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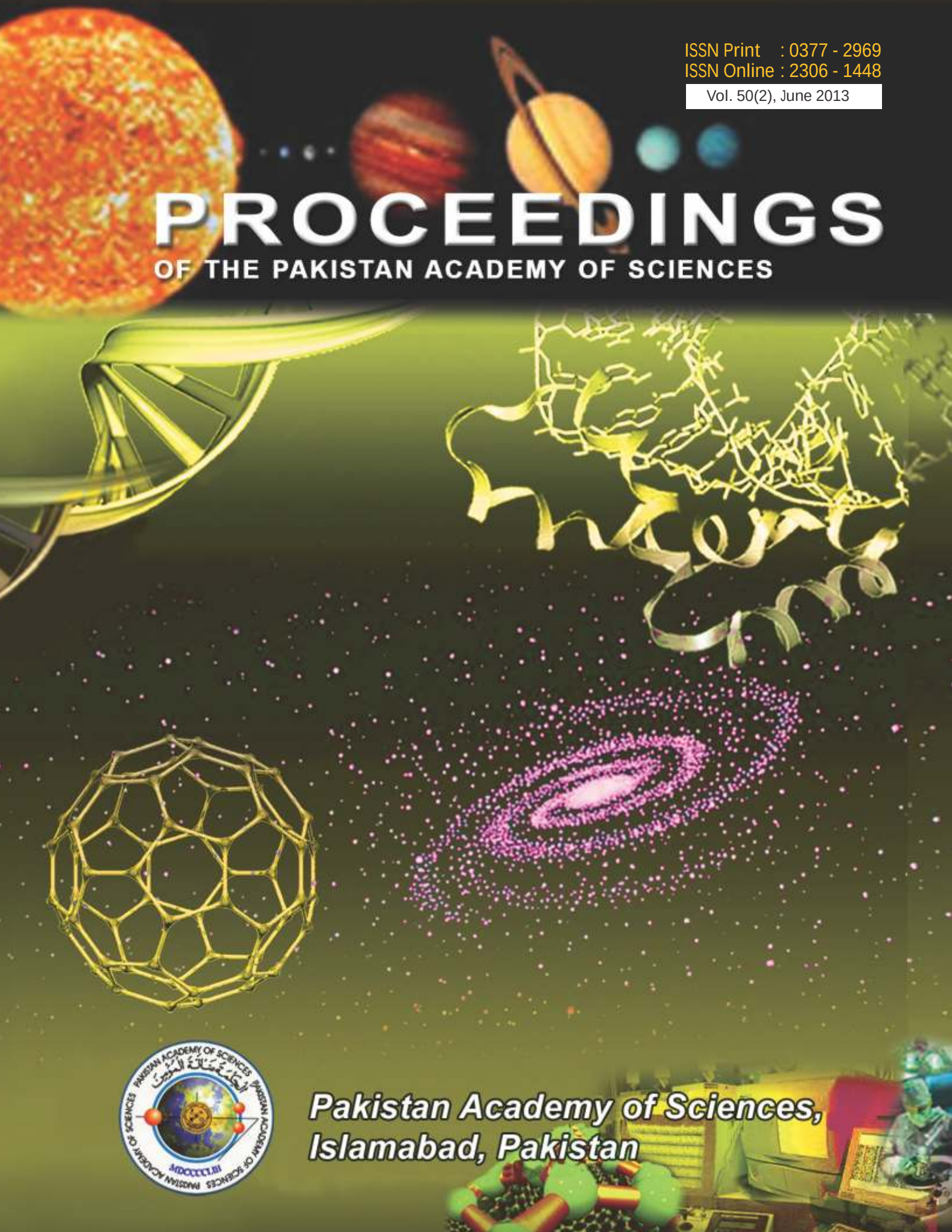

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CONTENTS

Volume 50, No. 2, June 2013 **Page**

Research Articles

Engineering Sciences

Parabolic Trough Solar Concentrators: A Technology which can Contribute towards Pakistan's Energy Future 99
— *Rizwan Masood*

Naturally Fractured Reservoirs – Yet an Unsolved Mystery 109
— *Muhammad Khurram Zahoor and Muhammad Haris*

Classification of Scientific Publications using Swarm Intelligence 115
— *Tariq Ali , Sohail Asghar, Naseer Ahmed Sajid and Munir Ahmad*

Life Sciences

Evaluation of Relative Efficacy of Hypertonic Saline and Lactated Ringer's Solutions in Experimentally-induced Hypo-volumic Shock in Dogs 127
— *Muhammad A. Zafar, Nemat Ullah, Imran Sarfaraz and Fahad Raza*

Comparative Phycoremediation of Sewage Water by Various Species of Algae 131
— *Farooq Ahmad, Amin U. Khan and Abdullah Yasar*

Physical Sciences

Remarks on (1,2)*-αĝ-Homeomorphisms 141
— *T. Salai Parkunan, S. Pious Missier, Ravi and A. Pandi*

Seasonal Variability in Virtual Height of Ionospheric F_2 Layer at the Pakistan Atmospheric Region 151
— *Akbar Ali Jilani, M. Ayub Khan Yousuf Zai, Faisal Ahmed Khan Afridi, Khusro Mian and Syed Nazeer Alam*

Extreme Rainfall Incidents over Sindh Province, against Different Return Periods 159
— *Muhammad Ali and M.J. Iqbal*

Properties of Univalent Solution for Complex Fractional Differential Equation 167
— *Rabha W. Ibrahim and Maslina Darus*

Obituary: Prof. Dr. Mustafa Shameel 173

Instructions for Authors 175

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Pakistan Academy of Sciences

Research Article

Parabolic Trough Solar Concentrators: A Technology which can Contribute towards Pakistan's Energy Future

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Abstract: The utilization of solar thermal energy has got prime importance in Pakistan due to the current energy scarcity and escalating cost scenario in the country. Parabolic Trough Solar Concentrator is one of the most reliable technologies for utilization of solar thermal energy. In solar thermal power generation, Parabolic Trough Solar Concentrators are most successful as almost 96 percent of total solar thermal power is generated across the world by utilizing this technology. Its high reliability, operational compatibility, comparative low cost and high efficiency adds to its high value among other resources. Fortunately, Pakistan lies in the high Solar Insolation Zone; thus, a huge potential exists to benefit from this technology. This technology may cater to the Pakistan's seasonal increased electricity demand. Apart from electric power generation, this technology may also have cost-effective solutions for Pakistan's other industries, like steam generation, preheating of boiler make-up water, air-conditioning, and hot water production for food, textile, dairy and leather industries. However, economic justification of such projects would be possible only on accomplishing an indigenous technology base. Globally, this is a proven technology, but in Pakistan there is hardly any development in this field. In this study, an effort has been made by designing and fabricating an experimental Parabolic Trough Solar Water Heater by utilizing locally available materials and manufacturing capabilities. On achieving encouraging results, a solar boiler (steam generator) is proposed to be manufactured locally.

Keywords: Pakistan's energy future, solar thermal energy utilization, solar concentrators

1. INTRODUCTION

A solar concentrator captures sunlight over a large aperture area and concentrates this energy onto a much small receiver area, multiplying intensity of the solar radiation by a concentration ratio in the range of 10–80. It is the process of concentration that allows troughs to deliver high temperature thermal energy. However, to achieve such concentration, a trough tracks the sun in one axis continually throughout the day.

A Parabolic Trough Solar Concentrator is a one-dimensional parabola that focuses solar beam (parallel) radiation onto a line. Physically, this line is a pipe (absorber) with a flowing liquid inside that absorbs the heat transmitted through the pipe wall and delivers it to the thermal load. Primarily, the heat transfer fluid (some high boiling-point liquid) is heated by passing it through the concentrator's absorber tube and then this heated fluid exchanges absorbed heat with some thermal load, mostly with water, to make steam that can run the steam turbine to produce electricity.

2. COMPARISON WITH OTHER SOLAR ENERGY UTILIZATION TECHNOLOGIES

Parabolic Trough Solar Concentrator is one of the most reliable technologies for utilization of solar thermal energy. There are many power plants running on solar energy across the world by using parabolic trough solar concentrators. After the successful commissioning of a huge commercial power plant in 2007 (68 MW) in Nevada, USA, this technology got a exponential growth, as currently

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almost 1700 MW of electricity is being produced by utilizing this technology and by the end of 2014 it will reach 3800 MW by the completion of under-construction projects [9, 10].

2.1. Comparison with Photo-Voltaic Technology

Both Photo-Voltaic (PV) and Concentrated Solar Panel (CSP) technologies are being used to generate electricity by utilizing solar energy but CSP is more reliable as compared with photovoltaic power generation. Solar energy is available only in day time but energy demand is for 24 hours a day. So, energy storage provides an option to increase grid reliability and there are many storage options available or under development. The main difference between CSP and PV technologies is energy storage efficacy. The ability of CSP to utilize high-efficiency thermal energy storage (TES) makes its more reliable technology (Fig. 03). The addition of TES produces additional value by shifting solar energy to periods when sun is not available. This enables CSP systems to supply solar-generated electricity during periods of cloudy weather or at night.

2.2. Comparison with other Solar Thermal Technologies

Parabolic Trough Solar Concentrators are most successful in solar thermal technologies. It is because of high reliability, operational compatibility, low cost (comparatively) and high efficiency as parabolic troughs are being utilized in different applications around the world. There are many power plants running on solar thermal energy, across the world and producing almost 1750 MW of electricity out of which 1682 MW (96% of total) are being produced by using parabolic trough solar concentrators [9, 10].

3. PROSPECTS OF THE TECHNOLOGY IN PAKISTAN

Currently, the country is experiencing a severe energy crisis, and its energy requirement over time is increasing rapidly. Over the past 15 years, the primary energy consumption in Pakistan has grown by about 80%, i.e., from 34 million tons oil equivalent (MTOE) in 1994-95 to 63 MTOE in 2009-10. The energy requirement is expected to be doubled in the next few years; by 2015 it is likely to cross 120 MTOE and by 2030 the requirement would reach 360 MTOE [8]. The country's primary energy supply currently comes mainly from fossil fuels, which is 88% of energy mix (47.5% indigenous natural gas, 0.7% LPG, 30.5% imported oil and 9.2% of energy mix). The rest 10.9% comes from hydropower and 1.2% from nuclear power [7]. Natural resources haven't been utilized effectively due to which our natural gas reservoirs are depleting and on the other hand oil prices in international market is also increasing.

Pakistan's current energy scenario demands extensive utilization of renewable energy resources in a cost effective manner. At present, hydropower is the only renewable energy resource, being utilized partially, and a 48 MW wind power project is in the erection phase as yet; all other renewable resources in the country are almost untapped.

Utilization of solar energy has got prime importance in current energy scarcity and cost inflating scenario. Parabolic Trough Solar Concentrator is one of the most reliable technologies in solar thermal technologies and developed countries are benefiting by utilizing this proven technology. Pakistan's geographical position & climate favors this technology but Economical justification for utilization of this technology may only be possible by accomplishing an indigenous technology base. Manufacturing, installation and operational excellence in this technology may contribute significantly in Pakistan's energy future.

3.1. Geographical Position and Solar Radiation Data

Pakistan's geographical position and climate has made this technology one of the reliable energy options as Pakistan lies in high Solar Insolation Zone and because of this there is a huge potential to get benefitted from this technology. Pakistan lies in between 25°-37° latitude which is one of most favorable regions for solar thermal energy utilization. In the fig.4 the regions in dark orange color are high Direct Solar Insolation (5.0-6.5kWh/m2/day) regions and most area of Pakistan lies in this region.

If we see the average monthly solar radiation

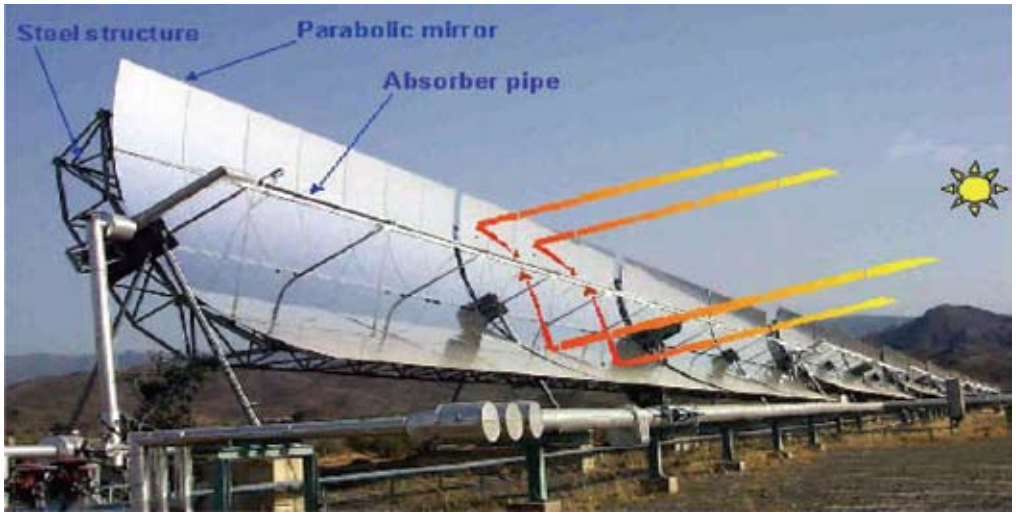

Fig.1. Schematic of a parabolic trough [2].

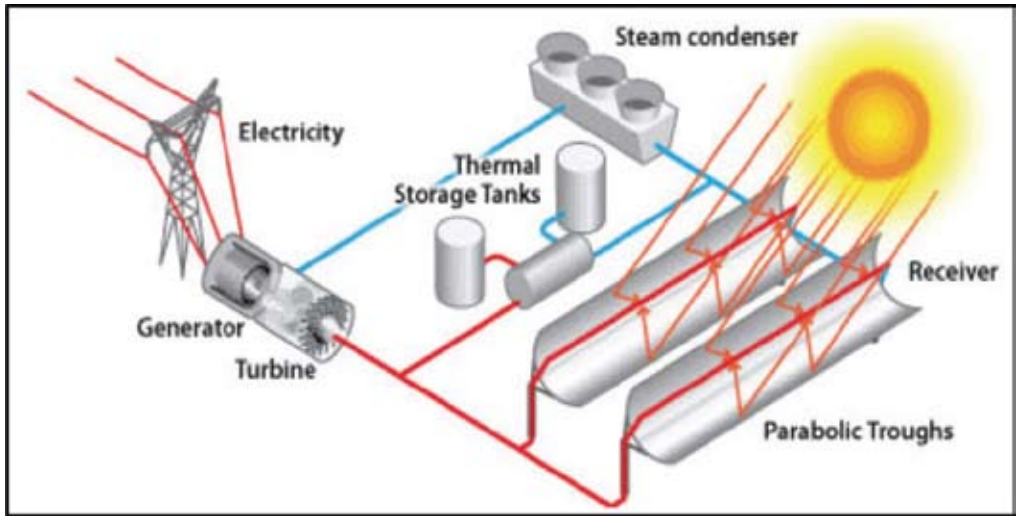

Fig. 2. Schematic of solar-thermal power plant [5].

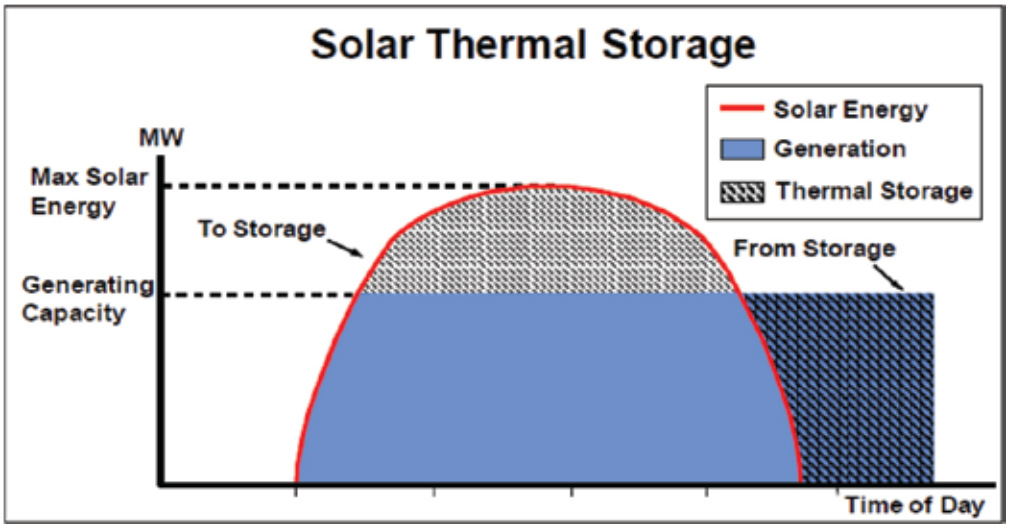

Fig. 3. TES extends the power production period [4].

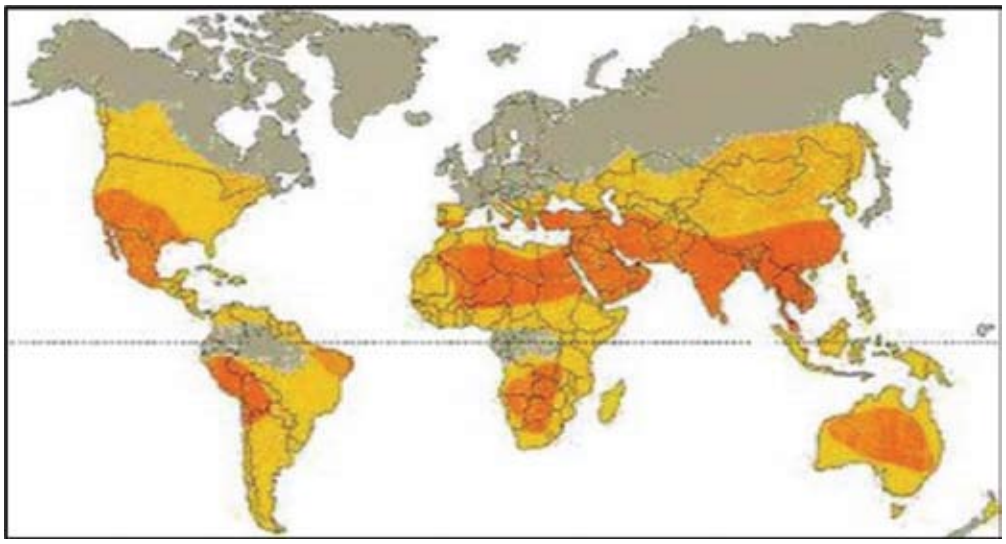

Fig. 4. Map for Solar Insolation region-wise [2].

data and day length for Lahore (Table.1) it can be observed that for the six months (i.e. Apr, May, Jun, July, Aug and Sep) high solar flux is available for more than 12 hr daily and these are the most troubled months for power shortage in Pakistan.

3.2 Applications for Pakistan's Local Industry

Parabolic trough solar concentrators have many cost effective solution for Pakistan's local industries, in addition to steam and power generation [6]. This technology may be utilized in:

i. Preheating of boiler make-up water.
ii. Utility hot water production in foods industry.
iii. Utility hot water production in dairy industry, as hot water is extensively used in dairy industry for cleaning purpose.
iv. Hot water production for leather industry (tanneries)
v. Hot water production for textile industry.
vi. In air-conditioning.

3.3. Socioeconomic Benefits

Solar thermal technologies have great potential to benefit our nation. They can not only diversify our energy supply by reducing our dependence on natural gas and imported fuels but also improve the quality of the air we breathe and offset greenhouse gas emissions. It can also stimulate our economy by low cost power generation and creating new jobs

Table 1. Monthly average radiation data on horizontal surface for Lahore [3].

Month	Day length (hr)	*Rad (cal/cm^2.day)*	*Rad (cal/m^2.day)*	*Rad (J/cm^2.day)*	*Rad (j/m^2.day)*	*Flux (W/m^2)*
JAN	10.2	256	2560000	1075.2	10752000	292.81
FEB	10.9	333	3330000	1398.6	13986000	356.42
MAR	11.8	435	4350000	1827	18270000	430.08
APR	12.8	499	4990000	2095.8	20958000	454.81
MAY	13.6	545	5450000	2289	22890000	467.52
JUN	14	533	5330000	2238.6	22386000	444.16
JUL	13.8	491	4910000	2062.2	20622000	415.09
AUG	13.1	448	4480000	1881.6	18816000	398.98219
SEP	12.2	436	4360000	1831.2	18312000	416.93989
OCT	11.2	361	3610000	1516.2	15162000	376.04167
NOV	10.4	297	2970000	1247.4	12474000	333.17308
DEC	10	244	2440000	1024.8	10248000	284.66667

in the manufacturing, installation and operations of solar energy systems.

4. TECHNOLOGY INCUBATION (EXPERIMENTAL RIG)

A parabolic trough concentrator system was designed and fabricated on experimental scale, for hot water production, under the research chair of SNGPL, at Chemical Engineering Department, University of Engineering & Technology, Lahore under the supervision of Drs. A.R. Saleemi and A.H. Delawari.

4.1. Parts, Construction and Materials

The experimental setup (Fig. 5) mainly consists of following components:

i. Reflecting surface
ii. Absorber
iii. Support structure
iv. Tracking system
v. Circulation system

The experimental rig was fabricated by utilizing locally available materials and local manufacturing capabilities. Brief description of components is as follow.

4.1.1 Reflecting Surface

It is the one of the most crucial parts of solar concentrator as efficiency of concentrator greatly depends on the reflectivity of reflector material.

Stainless steel sheet was used for this purpose. Although the value of reflectivity is not very good, however results obtained were encouraging (Fig. 06).

4.1.2 Absorber

Absorber is basically a metallic tube, which is usually coated black to enhance absorbance. The tube is jacketed with glass to reduce the convective heat losses. Absorbed solar radiation delivers heat energy to fluid moving inside pipe.

Copper tube has been used for the purpose. Absorber tube was fixed in glass jacket by the means of Teflon rings and mounting ends. The system was not evacuated due to cost constrains (Fig. 07).

4.1.3 Support Structure

This is the structure required to hold and support the absorber/reflector assembly at correct positions. It must be moveable in horizontal axis so that concentrator may track sun throughout the day. For this purpose trough was mounted on stand structure with the help of bearing (Fig. 08).

4.1.4 Tracking System

To achieve continuous delivery of concentrated solar energy, trough tracks the sun in one axis continually throughout the day.

Timer based tracking system has been used. It consists of two analog timers, on/off switches, electric motor and worm gear arrangement (Fig. 09). The system was calibrated with the movement of sun and the time based tracking was achieved.

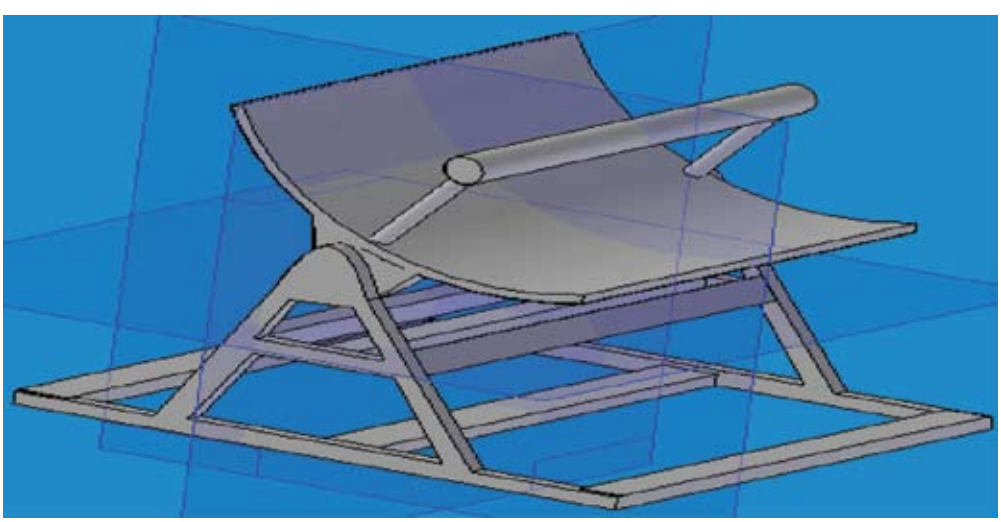

Fig. 5. Three dimensional model of the experimental rig.

Fig. 6. Reflecting Surface of the experimental rig.

Fig. 7. Absorber of the experimental rig.

Fig. 8. Support Structure of the experimental rig.

Fig. 9. Tracking system structure of experimental rig**.**

4.1.5 Circulation System

Circulation system is a wide term, used here for complete circuit of heat transfer fluid including pumps hot fluid reservoir and circulation pattern.

Centrifugal pump was used along with rotameter, online thermometer and valve arrangement for in circulation system (Fig. 10).

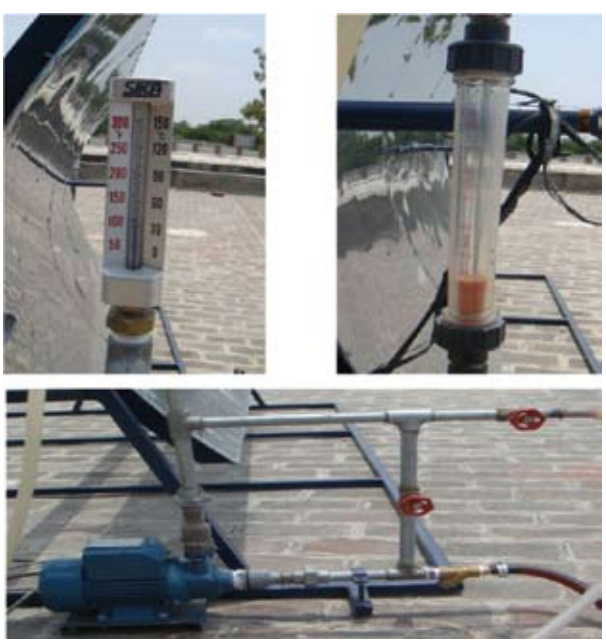

Fig. 10. Circulation System of the experimental rig.

4.2. Dimensions of the Experimental Rig

The fabricated experimental rig has the following dimensions

Length of trough, L = 2.133 m
Length of aperture, L_a = 1.1 m

Aperture area, A_{ap} = 2.133 x 1.1 = 2.346 m^2

Outside dia of glass envelop, D_{og} = 0.033 m

Inside dia of glass envelop, D_{ig} = 0.03 m

Outside dia of absorber tube, D_{or} = 0.014 m
Inside dia of absorber tube, D_{ir} = 0.013 m

Outside surface area of absorber tube, A_{or} = .Л Dor L = 0.0938 m^2

Inside surface area of absorber tube, A_{ir} = Л Dir L = 0.087 m^2

Outside surface area of glass envelop , Aoe = .Л Doe L = 0.221 m^2

Now the Concentration Ratio for these dimensions may be calculated as:

C.R. = Aperture area / Absorber area = A_{ap} / A_{or}

C.R. = 25.0106 = 25

4.3. The Experiment and Its Results

An experiment was designed to study the performance of the fabricated experimental rig. As the rig have two temperature sensors, one on outlet and other on inlet of absorbing tube and no heat storage was available so the focus was on investigating the change in temperature of circulating water in one pass at different flow rates. The flow rate was optimized on the basis of heat transfer coefficient.

Volumetric flow rate of water,

F_w = 75 L/hr

= 2.08 x 10^{-5} m^3/sec and

Mass flow rate, m_0 = 0.0206 kg/sec

Water was circulated by means of pump and the flow rate was adjusted with the help of bypass valve on the rig. Temperatures of both points, inlet and outlet of absorber, were noted. The experiment was carried out when the direct solar radiation intensity was 480.5 W/m^2. The increase in temperature in one pass achieved was 9°C and in multi-pass

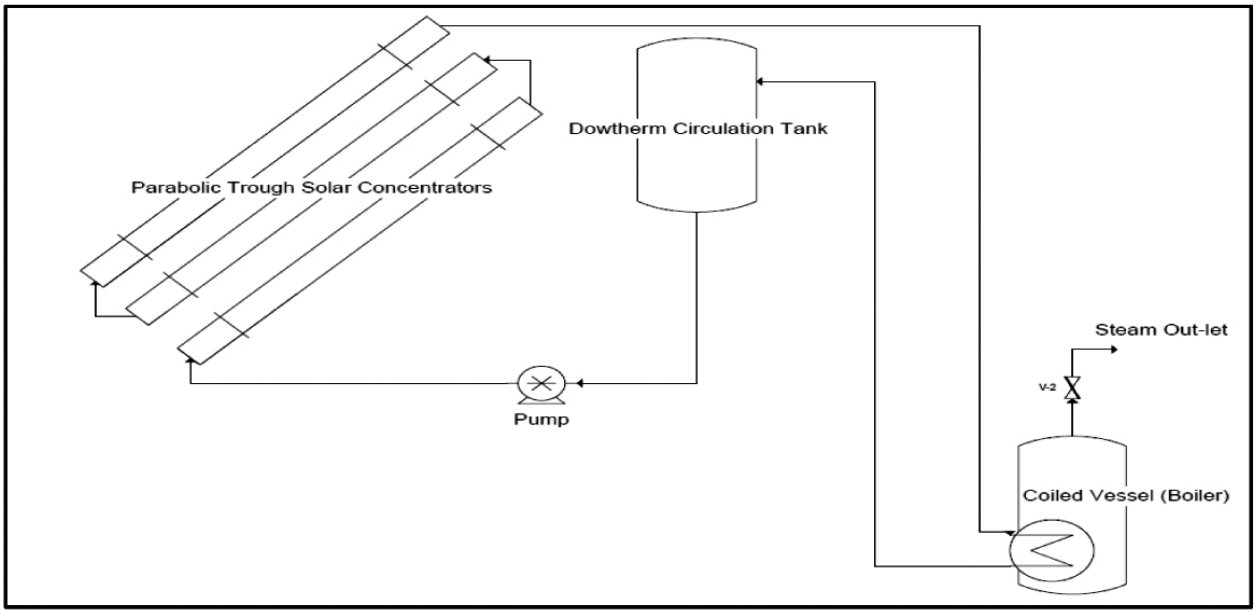

Fig. 11. Schematic of the proposed solar boiler.

arrangement 85°C temperature of water reservoir was archived.

4.4. Efficiency and Performance

Overall efficiency of a parabolic trough solar concentrator is based on two types of performance: (i) optical performance; and (ii) thermal performance.

4.4.1 Optical Performance

Absorbed radiation per unit area of un-shaded aperture is given by:

$$S = I_b \rho(\gamma\tau\alpha) K_{\gamma\tau\alpha} \quad [1]$$

Where I_b is effective incident beam radiation on the plane of aperture, ρ is the reflectance of the concentrator, γ is intercept factor, τ is transmittance, and α is absorptance. $K_{\gamma\tau\alpha}$ is an incidence angle modifier that can be used to account for deviations from the normal of the angle of incidence of the radiation on the aperture.

4.4.2 Thermal Performance

$$Q_u = A_a F_R \left(S - \frac{A_r U_L}{A_a}(T_i - T_a)\right) \quad [1]$$

where A_a is the un-shaded area of the concentrator aperture and A_r is the area of the receiver, S is the absorbed solar radiation per unit of aperture area, T_i and T_a are inlet fluid temperature and ambient temperature F_R is the collector heat removal factor.

$$F_R = \frac{m_o Cp}{A_r U_L}\left(1 - \exp^{\left(-\frac{A_r U_L F}{m_o Cp}\right)}\right) \quad [1]$$

4. THE WAY FORWARD

To get our nation benefited by this technology, we must have to develop technical expertise in manufacturing and operations of Parabolic Trough Solar Concentrator systems. Initially, we should focus on commercialized thermal application especially for water heating and hybrid (fossil fuels-CSP) steam generation. This will make us able to run infrastructure level projects.

Development of a Parabolic Trough Solar Boiler is purposed (Fig. 11). The heating mechanism of purposed setup is indirect heating i.e. primarily the heat transfer fluid (some high boiling point liquid) will be heated, by passing it through concentrator's absorber tube, and then this heated fluid passes through coiled heat exchanger and exchanges its heat with water and water will make steam that can run steam turbine.

5. ACKNOWLEDGEMENTS

The author is thankful to SNGPL Research Chairs, Dr. Anwar Rasheed Saleemi and Dr. Amjad Hussain Delawari, for providing me this opportunity to materialize the idea. I am also grateful to Dr. Amjad Hussain Delawari for his technical, administrative and

moral support and to Dr. Naveed Ramzan for his support and guidance.

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Research Article

Naturally Fractured Reservoirs – Yet an Unsolved Mystery

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Abstract: Some of the world's most profitable reservoirs are assumed to be naturally fractured reservoirs (NFR). Effective evaluation, prediction and planning of these reservoirs require an early recognition of the role of natural fractures and then a comprehensive study of factors which affect the flowing performance through these fractures is necessary. As NFRs are the combination of matrix and fractures mediums so their analysis varies from non-fractured reservoirs. Matrix acts as a storage medium while mostly fluid flow takes place from fracture network. Many authors adopted different approaches to understand the flow behavior in such reservoirs.

In this paper a broad review about the previous work done in naturally fractured reservoirs area is outlined and a different idea is initiated for the NFR simulation studies. The role of capillary pressure in natural fractures is always been a key factor for accurate recovery estimations. Also recovery through these reservoirs is dependent upon grid block shape while doing NFR simulation. Some authors studied above mentioned factors in combination with other rock properties to understand the flow behavior in such reservoirs but less emphasis was given for checking the effects on recovery estimations by the variations of only fracture capillary pressures and grid block shapes. So there is need to analyze the behavior of NFR for the mentioned conditions.

Keywords: Naturally fractured reservoirs, capillary pressure, grid block shape

1. INTRODUCTION

Naturally fractured reservoirs (NFR's) have great importance and differ from conventional reservoirs as these contain fractures throughout the reservoir. These reservoirs usually have low matrix permeability and high fracture permeability having oil or water wet rock properties which may results in low recovery. So good understanding of flow behavior is required to optimize recovery process. Analysis and simulation of fractured reservoirs are done by dividing such reservoirs into matrix and fracture systems. Initially dual porosity concept was introduced but now a day's dual porosity and dual permeability approach for performing NFR simulation studies is widely accepted.

2. LITERATURE REVIEW

From 1960's naturally fractured reservoirs acquired great importance for recovery considerations. Barenblatt et al [1] firstly introduced the dual medium model for the simulation of naturally fractured reservoirs. Analysis was done by assuming that the total fluid flow is a combination of flow both from matrix and fracture. In 1963, Warren and Root [2] worked on the naturally fractured systems and by assuming primary porosity as a storage medium of fluid having very low flowing capacity, an idealized model was developed for analysis of fluid flow such reservoirs . During this flow, matrix blocks continuously feed the fracture system but fluid cannot move directly from one matrix to other matrix

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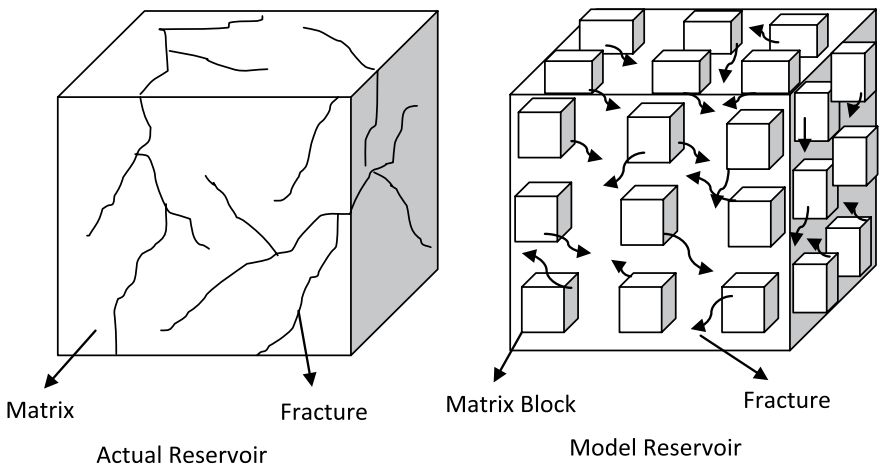

Fig. 1. Idealized fractured system.

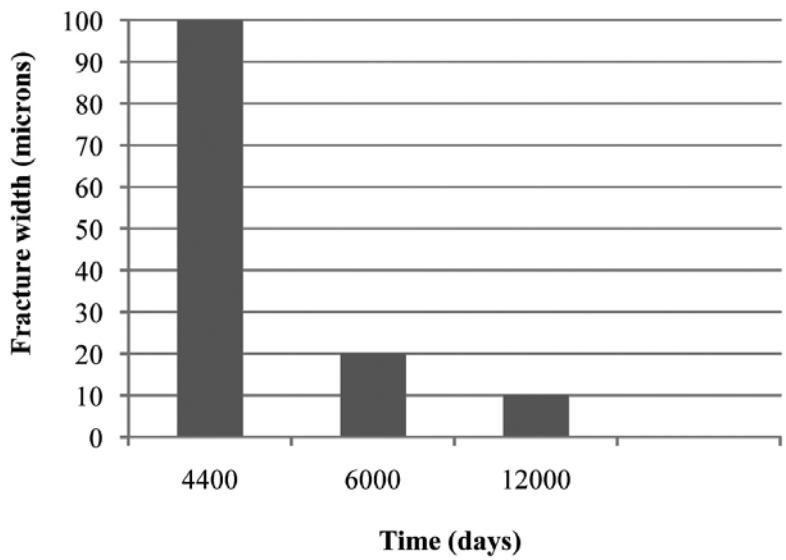

Fig. 2. Time taken to achieve total recovery for different fracture widths.

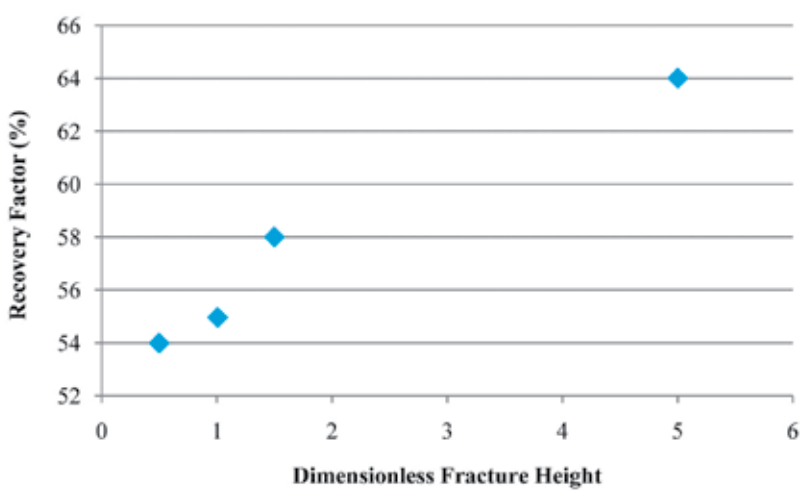

Fig. 3. Recovery factor for different dimensionless fracture height.

block (Fig. 1). Unsteady state flow and pressure built up performance had also been examined and suggested that two parameters, interporosity flow parameter (λ) and fluid storativity ratio (ω) are important for the NFR simulation studies.

In 1976, Reiss [4] worked to develop understanding about fractured reservoirs. He studied different fracture parameters such as facture permeability, fracture width, fracture porosity etc, depending upon different geometrical systems in reservoirs. He also highlighted different techniques which can be used for the identification of fractures in reservoir. Van Golf Racht [6] in 1982 published a book which comprised of detailed analysis of fracture characteristics. He discussed different fracture properties like fracture permeability and capillary pressures which exists in such reservoirs and concluded that capillary forces may contribute or may oppose fluid flow during imbibitions and drainage process. He suggested that accurate knowledge of matrix and fracture properties is necessary to get truthful results while simulating naturally fractured reservoirs. Saidi [7] suggested that due to capillary effects in vertical fractures and wettability, capillary pressure in fractures could exist. He also recommended that due to blocks interactions, capillary re-infiltration phenomenon which is the fluid flow from upper to lower blocks should be considered for better simulations results.

In 2007, Jadoon et al [13] investigated the carbonate fractured reservoir in *Kohat/Potwar* region, Pakistan. They discussed the problems during the identification of natural and induced fractures, categorized the fractures into conductive, resistive, systematic and unsystematic based on borehole imaging tool and recognized the impact of fractures on production.

3. SIMULATION STUDIES

In 1966, Romm [3] was the first person who performed the experimentation studies to analyze fluid flow in fractured media. His findings showed that a linear relationship exists between phase relative permeability and phase saturation while considering zero fracture capillary pressure. Gilman and Kazemi [5] extended the work of Warren and Root [2] by introducing a shape factor, as given in equation (1). They worked on dual porosity simulator by giving more importance to gravity and capillary interactions. They proposed that no particular method exist which can determine facture capillary pressure and also maximum fracture and matrix capillary pressure must be equal for balanced gravity and capillary forces.

$$\sigma = \frac{1}{4}\left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right) \tag{1}$$

In 1990, Firoozabadi and Hauge [8] analyzed capillary pressure in naturally fractured reservoirs. They proposed a model which relates the existence of capillary pressure in fractures and phase saturation by considering two fracture parameters, aperture and roughness. Through experimental work they indicate the presence of non-zero capillary pressure in fractures. In 2001, Akin [9] analyzed fracture network system with the help of history matching experimental data and numerical model to get the fracture relative permeability. During his analysis, he used different matrix and fracture capillary pressures combinations to obtain the best production and pressure history match. De la Porte et al [10] in 2005 did their analysis on capillary pressures in fracture network system. Two different fluid injection scenarios were analyzed by considering different fracture properties like fracture width and dimensionless fracture height and examined the variations on recovery. Results shows that total recovery is achieved in early times when fracture width is increased and maximum recovery took place for greatest dimensionless fracture height (Fig. 2, 3).

Qasem et al [11] investigated capillary imbibition effects on recovery in partially fractured reservoirs. During analysis, they implemented low and high water injection scenarios and concluded that generally fracture intensity (FI) has a direct impact on recovery as it decreases with low FI for a particular high water injection rate and acts inversely for reverse conditions. They also proposed that counter-current capillary imbibition phenomena occur for high fracture intensity reservoirs during low water injection rates which cause an increase in recovery due to higher areal sweep efficiency and later breakthrough and co-current capillary

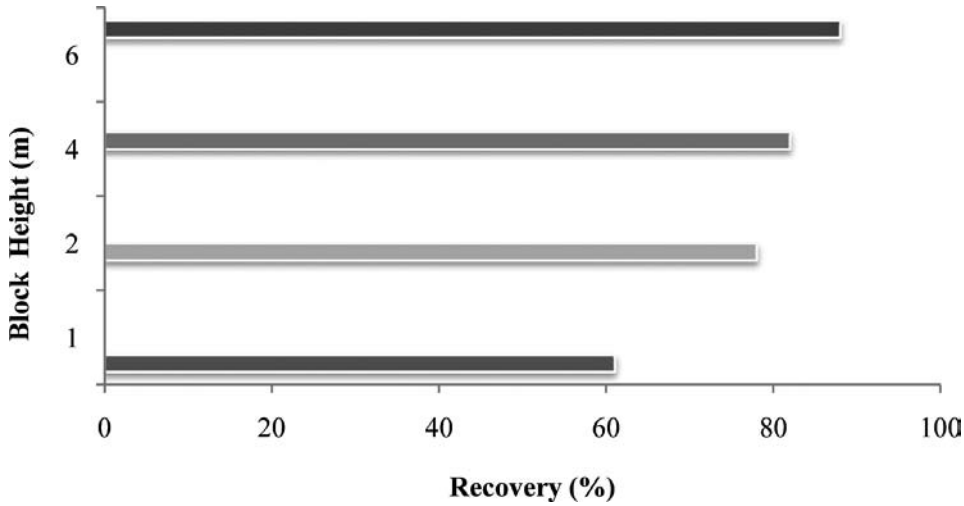

Fig. 4. Ultimate recovery achieved for different block height.

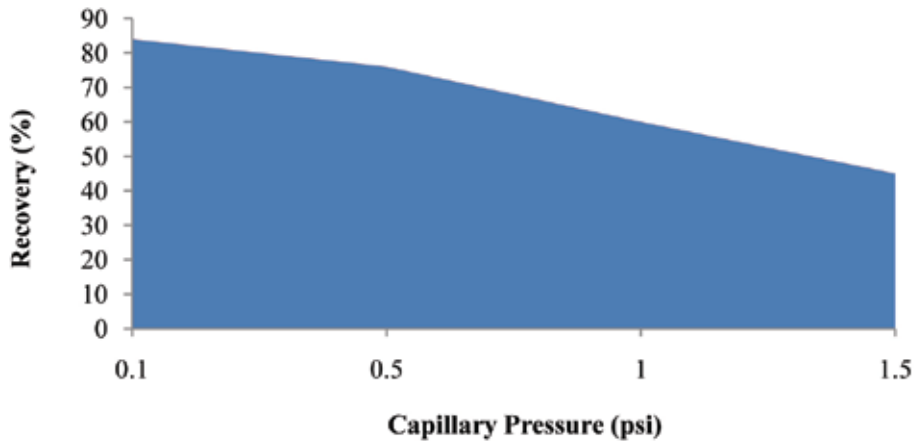

Fig. 5. Inverse relation existing between recovery and capillary pressure.

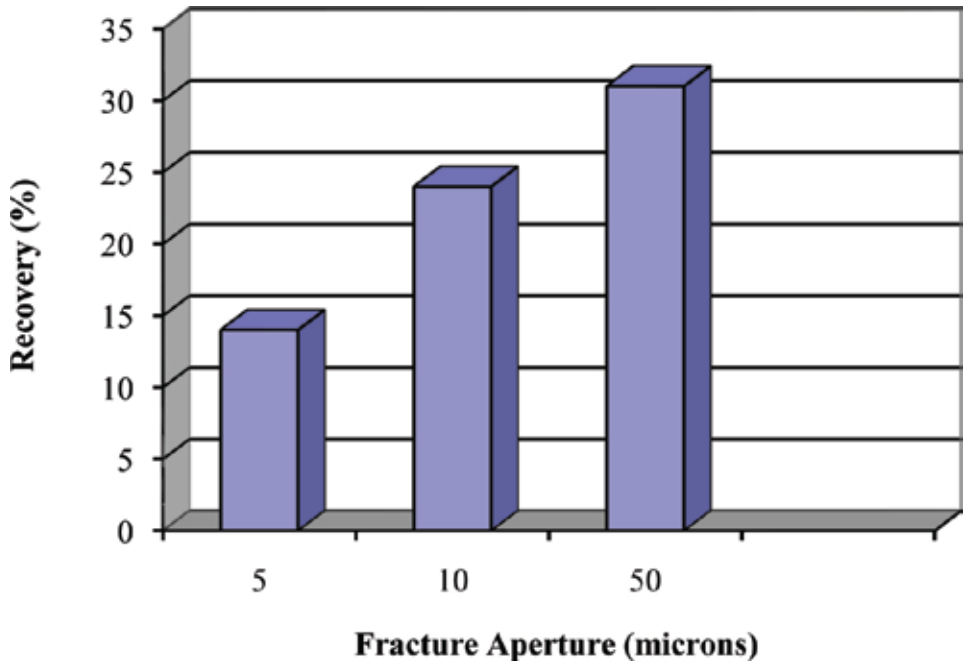

Fig. 6. Comparison of field oil recovery for different fracture widths.

imbibition phenomena occur on low fracture intensity reservoirs thus decreases recovery. Sharait et al [12] studied NFR for oil/gas system having immobile water. They accepted that re-infiltration term effects on production rate but it could not change ultimate recovery. They also suggested that because of matrix block interactions recovery acts as independent from vertical fracture permeability and an early ultimate recovery is achieved when fracture capillary pressure is increased. By increasing the block height oil recovery is increased and an inverse trend exists between matrix capillary pressure and oil recovery (Fig. 4, 5). The reason is that when capillary force increases in matrix less oil will drain to fracture.

In 2007, Tajdar et al [14] in their work highlighted the importance and effects of matrix sub-gridding on recovery in naturally fractured reservoirs. They utilized the technique of dividing a single matrix block into several subgrids so that a more accurate transmissibility of fluid between matrix and fractures could occur. They analyzed different scenarios and concluded that in some cases there could be an error in recovery estimation if improper sub-gridding of matrix block is done. In their analysis variation of fracture properties like fracture capillary pressures are not being discussed.

Ulebergand Kleppe [15] in 2010 did their analysis for the simulation of fractured reservoirs based on physical characteristics of reservoir and fluid flow mechanism in them. They showed that capillary continuity of blocks increase the ultimate recovery. They emphasized to model fractured reservoirs on the source of multiple grid concepts for better results. From this study it can be concluded that the existence of capillary pressure in the fracture system will also affect the recovery. In 2010, Noroozi et al [16] investigated the similar sort of work done by De la porte [10] by considering instead of single porosity single permeability model to dual porosity dual permeability concepts. He focused his work on water and gas injection scenarios while simulating the fractured reservoirs by changing the capillary pressures, fracture width and fracture height. (Fig. 6) shows that when fracture aperture is increased from 5 to 10 microns, recovery is almost doubled.

In 2010, Kiasari et al [17] studied the effects of rock fluid properties during SAGD production and showed that capillary pressure helps in steam penetration from fractures into matrix blocks under water wet condition and act vice versa in oil wet systems. Also an increased early production occurred when capillary pressure is increased from zero. Famian and Masihi [18] in 2010 studied the influence of different properties on recovery while applying immiscible gas injection and during natural depletion. Results indicated that recovery is much affected by matrix threshold capillary pressure and matrix block height.

In 2010, Lemonnier and Bourbiaux [19] described the brief history about the simulation of naturally fractured reservoirs. They discussed different driving mechanism which effect on recovery in fractured reservoirs. In this paper, the use of dual porosity simulator is also been discussed in detail. Jabbari and Zeng [20] in 2011 analyzed the behavior of naturally fractured reservoirs under stress sensitive conditions. Production creates the pressure depletion which may results in changing the characteristics of fractures. So they proposed a model which includes the use of geo-mechanical and fluid flow factors while simulating NFR. The effect of stresses on pressure build up and drawdown curves is also discussed.

4. CONCLUSION AND RECOMMENDATIONS

This study signifies the importance of fracture capillary pressure and grid block shape in recovery estimations. As a conventional practice, the recovery estimations have been made by changing fracture properties such as fracture width, dimensional fracture height, etc while keeping the same grid block shape (mostly cubical). Furthermore, facture capillary pressure effects are not incorporated while conducting simulation studies, which may lead to inaccurate estimations. So, there is an immense need of study to analyze the effects of varying fracture capillary pressure from zero to other numeric values while keeping the same other fracture properties. In addition, the variations in the grid block shape other than cubical must also be analyzed for better representation of reservoirs while numerical modeling for recovery estimations. Consequently,

the conducted numerical or simulation studies can lead to improved recovery estimations and, thus, result in better field development planning and economical analysis.

5. NOMENCLATURE AND ABBREVIATIONS

NFR Naturally fracture reservoir
FI Fracture intensity
SAGD Steam assisted gravity drainage
L_x Length of matrix block in x-direction
L_y Length of matrix block in y-direction
L_z Length of matrix block in z-direction
σ Shape factor

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Research Article

Classification of Scientific Publications using Swarm Intelligence

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Abstract: Document classification is an important task in data mining. Currently, identifying category (i.e., topic) of a scientific publication is a manual task. The Association for Computing Machinery Computing Classification System (ACM CCS) is most wildly used multi-level taxonomy for scientific document classification. Correct classification becomes difficult with an increase in number of levels as well as in number of categories. Domain overlapping aggravates this problem as a publication may belong to multiple domains. Thus manual classification to taxonomy becomes more difficult. Most of the existing text classification schemes are based on the Term Frequency and Inverse Document Frequency (TF-IDF) technique. Similar approaches become computationally inefficient for large datasets. Most of the techniques for text classification are not experimentally validated on scientific publication datasets. Also, multi-level and multi-class classification is missing in most of the existing schemes for document classification. The proposed approach is based on metadata (i.e., structural representation), in which only the title and keywords are considered. We reduced the features set by dropping some of the metadata, like abstract section of the scientific publication that diversifies the result accuracy. The proposed solution was inspired from the well-known evolutionary Particle Swarm Optimization (PSO). The proposed technique results in overall 84.71% accuracy on Journal of Universal Computer Science (J.UCS) dataset.

Keywords: Topic identification, category identification, document classification, multi-class, multi-level

1. INTRODUCTION

Classification is an important task in data mining [1]. Automated text categorization is becoming more important with the advent of digital libraries and with a rapid increase in the number of documents on the web. The research community is producing a large number of scientific documents. These documents are then searchable over the internet using search engines, digital libraries and citation indexes. There is a need to classify this huge amount of documents into a hierarchy or taxonomy [2]. Similarly, the document's relatedness to a node in an existing taxonomy will assist in searching the user-relevant information in an efficient way. Accuracy of information retrieval basically depends on accurate classification of the documents [3]. Besides information retrieval, accurate classification helps in analysing trends, finding expertise, and the relevant document recommender system.

Classification is a two level approach in which first level generates a model from the training set of instances and the second level checks accuracy of the classifier [4]. There are a number of approaches for document classification, such as Decision Tree [5], Naive Bays Classifier [6], Particle Swarm Optimization (PSO) [7], Support Vector Machine (SVM) [8], and Term Frequency and Inverse Document Frequcy (TF-IDF) [9]. We have already reported a detailed survey [10] towards automated text classification in the context of supervised learning.

One important step towards the document classification is the category identification. Currently, authors of scientific publications identify the relevant category or categories (from

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onward written as category/ies) to their papers manually. Common categorization used in research community is the Association for Computing Machinery Computing Classification System (ACM CCS) [11].

Manually, category identification for a document is difficult task for new researcher, especially if the work belongs to multiple domains. Due to the diversity in domains and mapping of one domain to another domain, the manual classification task is becoming extremely difficult. This research is an effort to bridge the gap between users towards identifying correct document category, and suggest possible categorization to the author's work automatically.

Accurate categorization can be helpful in relevant information (i.e., document) retrieval. Traditional text classification is usually attained by assigning a document to one class, but in scientific community document can belong to multiple categories. We propose that initially the document may be categorized in the top category, and after matching with the top category it may be further classified to its sub-levels as depecited in Fig. 1. The search space for new document clasification is reduced by considering the sub-levels of the parent category/ies selected at the first level. Similar approach may be adopted for the third level, if it exists for any category selected at first and second level.

Scientific document classification has structural advantage over the unstructured document classification, as structural representation increases accuracy of the classification [10]. Most of the existing text classification schemes are based on the TF-IDF technique [7, 12]. Similar approaches become computationally inefficient for large datasets. Detail survey towards document classification is given by Sebastiani [12]. Similarly, most of the techniques for text classification are not experimentally validated on scientific publication datasets. Also, multi-level and multi-class classification is missing in most of the existing schemes for document classification [7, 13-18].

The proposed technique is based on metadata (i.e., structural representation), in which we considered the *title* and the *keywords*. The *title* of a scientific publication normally reflect theme of the work and and the *keywords* are the representative features of the paper to their category/ies.

Every year large number of documents are added to the web. These documents contain a large number of attributes, on the basis of which accurate classification is becoming extremly difficult. Self-adaptability of evolutionary approaches makes it possible to use it for such a dynamic problem having many features for huge number of documents. A document can act as a particle with regard to its own position and the position of other documents in the taxanomy. Thus, based on their position, we can find the similarity between any two documents.

The proposed solution is inspired from the well known PSO algorithm [19, 20]. Documents in the taxonomy are represented with its local position in a category along with global position with neighbourhood categories documents. Classifying a new Test Document (TD) depends with the similarity measure of all particles in each individual category. At second and third level, the movement of new document in the taxonomy depends on the selection of category/ies at the first level. The movement of TD is inspired with the document's similarity in each individual category. Classification of a scientific publication to a taxonomy is a multi-level classification. The number of documents and the nature of classification (e.g., multi-class and multi-level) makes this problem more complex. Due to these two reasons, PSO stands out as one of the optimum solution. PSO is simple, easy to implement and computationally efficient.

We have implemented our proposed solution and tested it on the Journal of Universal Computer Science (J.UCS) dataset [21], in which *2/3* instances were used for the training set and *1/3* for the test dataset. Furthermore, we assigned some heuristics for the selection of system generated category. We implemented our proposed

techniques and concluded overall 84.71% accuracy on the J.UCS dataset.

Rest of the paper is organized in a way that Section 2 presents the problem statement and Section 3 contains related approaches with critical analysis, Section 4 presents the representation of scientific documents, Section 5 contains the proposed technique for the scientific publication classification, Section 6 contains the experimental results of the proposed approach with detailed discussion and analysis and Section 7 concludes the paper and provides future directions.

2. PROBLEM STATEMENT

Document classification assigns a new document (e.g.., a research paper) to a set of previously defined taxonomy. The pre-defined categories can be of any type. Normally, in computer science the most common categorization is the ACM Computing Classification System (ACM CCS 1998) on which we have tested our proposed approach. Formally, the problem can be defined as given in Eq. 1.

$$d_1 : D \in C_{j..k} : C = d_1 : C \ \dots\dots Eq: 1$$

Where *D* is a set of documents. *C* is a set of predefined categories. The problem is to classify a document *di* of type *D* to category/ies $c_{j..k}$ belonging to category set *C*. The problem is also depicted in Fig. 1. TD is the user provided publication, whereas *A to K* are the main categories of the ACM CCS. Each category is divided into sub-categories which have further sub-categories at third level. Relevant category for new document has to be calculated in the ACM CCS.

3. RELATED WORK

Different approaches for document classification based on Particle Swarm Optimization (PSO) [19, 20], Support Vector Machine (SVM) [8], Bayesian network [6] etc exist in literature. Some of the document classification techniques work on metadata, while others work on the complete text available in the document. Very few Classification techniques are available for the scientific publication category identification as compare to other document classification approaches. Scientific publication contains both metadata and text, which increases its classification accuracy. Some of the approaches towards document classification with reference to the multi-class and multi-level classification are analysed in detail.

Our survey towards the document classification in the context of supervised learning technique is given in [10]. Classification in both structured and unstructured context is analysed. With published literature [13] it is strongly argued that structured documents give more accuracy in classification over unstructured documents. This survey provides theoretical comparison of different techniques, while accuracy and efficiency of classifiers is missing [10].

A PSO based document classification for web documents is given in [7]. In this approach the documents are pre-processed by removing *stopwords* [22] and *wordsstemming* by using porter stemmer [23] algorithm. After preprocessing, the documents were represented as document term frequency matrix. Documents were finally represented as term vectors, using TF-IDF weighting approach [7]. Feature selection was done through entropy weighting scheme [25]. Entropy weighting scheme is done using local weighting of term *k* and global weighting of term *k* as *Ljk* × *Gk*. after feature selection, particle swarm optimization (PSO) was used as a classifier.

Initialization of individual particles is done randomly; the structure for each particle at given iteration is represented [7] as

$$\mathrm{X}_{\mathrm{i}}^{0} = (\mathrm{x}_{\mathrm{i}1}^{0}, \dots, \mathrm{x}_{\mathrm{in}}^{0})$$

Where *0* represents the iteration and n represents the term numbers in document set. The velocity of individual particles [7] is given as, which corresponds to the update quantity of all weighting values.

$$\mathrm{V}_{\mathrm{i}}^{0} = (\mathrm{v}_{\mathrm{i}1}^{0}, \dots, \mathrm{vi}_{\mathrm{in}}^{0})$$

Finally, the effectiveness is measured in terms of precision and recall, as:

F1 = (2 * Precision * Recall) / (Precision + Recall)

Experimental evaluations were performed on two standard text dataset reauter-21578 and TREC-AP. In this approach [7] no weighting mechanisms for structural contents are used. Similarly, classifying a document to multiple categories is missing. Strategies for multileveled classification are also not discussed.

Effectiveness of PSO with respect to different dataset towards classification problem is given in [15]. Ten different datasets with multiple instances, classes and number of parameters composing each instance are taken. PSO accuracy in terms of error rate is compared with other nine classifiers. On three data sets PSO outperformed than all other classifiers. PSO efficiency with increasing number of classes is highlighted, which may be due to implementation or similarity measure used for evaluating fitness function.

Improving the document classification with structural contents and citation based evidence is given in [16]. For classification, both structural (i.e., title and abstract) and citation based information is considered. Different similarity measures for both structural (i.e., bag of words, cosine, and Okapi) and citation based (e.g., Bibliographic coupling, Co-citation, Amsler, and Companion) similarity are used. Genetic programming is used for classification of new document. For prediction of new document, best similarity tree for each class is maintained. Class for the new document, is decided on the based of majority voting from each classifier.

A new approach based on the neighbourhood preserving embedding (NPE) with PSO is given in [17]. NPE preserves the local manifold structure and preserve the most discriminating features for classification. Documents features in the higher domain X are reduced to the lower domain Y by using the NPE. PSO is used similar to the approach presented in [7] Discriminative features extraction plays an important role in increasing the document classification accuracy. Results of the NPE with PSO has shown better results than LDA PSO, LSI PSO, and LSI-KNN [17].

Bayesian based approach for the classification of conference paper is given in [13]. 400 educational conference papers in four categories (e-learning, Teacher Education, Intelligent Tutoring System, and Cognition issues) are used for constructing the Bayesian network. Only keywords are used for conference paper classification. Compound keywords are parsed into Single keywords which are ranked with respect to frequency and top 7 keywords for each category are considered as input for Bayesian network. Each category shares some common keywords along with some individual keywords. The network has trained with 100 papers for each category. This technique is efficient due to the reason that only keywords are used for classification. Conversely, the misclassification error can increase due to non availibilty of keywords in some documents or due to wrong keywords assigned by the authors.

Text classification using swarm intelligence in terms of automated grouping of PDF documents is given in [18]. The algorithm presented is inspired from the ant colony optimization. Basically classification is used in terms of clustering; PDF documents are converted into text files. Relative frequency of words in a document is calculated which is normalized with the word frequency in all documents to lower the importance of words occurring in all documents. Cosine similarity is used as a measure between the two objects.. For convergence the picking parameter of ant for picking an object was reduced.

Association rule mining approach towards document classification is given in [26]. Associative rule mining discovers relationships among items in a dataset. Documents are represented as transactions. *Stopswords* removal and *Stemming* is used to reduce the transaction size. Initially, rules are generated using the apriori algorithm. Two methodologies are used for the rules generation: one is, rule generation for each category; and the other one is association rule mining for the categories collectively. On the basis of these rules, classifier is developed. Experimental results are presented on the Reuters-21578 text collection [27].

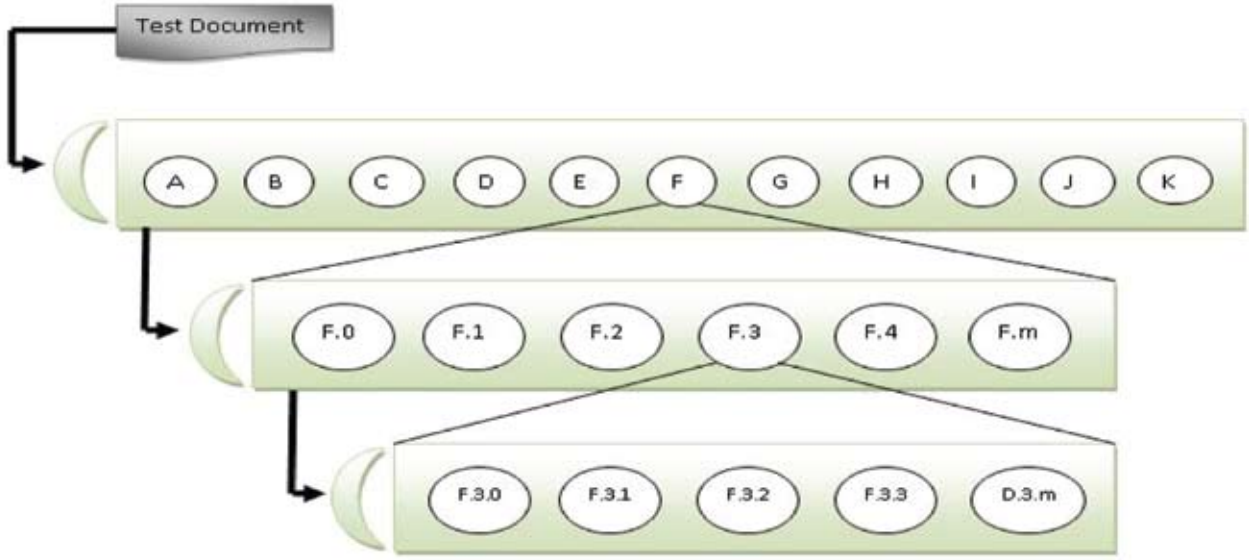

Fig. 1. Classifying new document to ACM CCS.

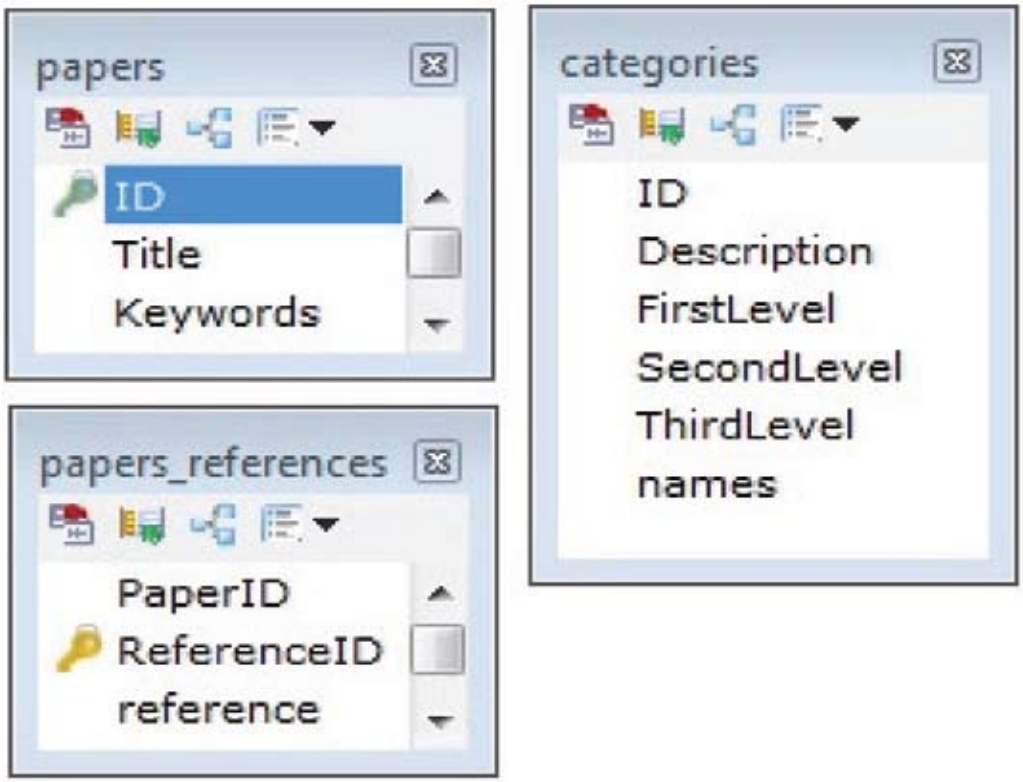

Fig. 2. Relevant information of documents.

Linear text classification using category relevance factors (CRF) is given in [14]. CRF is maintained for all documents that belongs to a category. Profile vector for each category is maintained from CRF feature vector. Based on the cosine similarity between papers and categories, the document is classified to the category having maximum similarity.

Structured document representation increases the classification accuracy, as scientific publications are well structured documents; therefore, it is necessary to classify them in taxonomy with high percentage of accuracy. Existing approaches lack in use of relevant information of both metadata data and text available in the document. Some approaches towards classification rely on either *keywords*, or *abstract* or *fulltext* of the document. Most of the existing schemes are not evaluated for scientific publications datasets. Only two approaches [13, 16] focus on the classification of scientific publications. Similarly, most of the schemes do not consider multi class classification and multi level classification. To overcome these limitations, we have devised an approach that not only uses the relevant information for classification but also deals with multi-class and multi-level classification. Detailed discussion on relevant data selection from scientific publication is presented in the following sections.

4. DOCUMENT REPRESENTATION

For efficient document classification metadata and contents can be used. The common features are *title*, *abstract*, *authors*, *keywords*, and *references*. In most of the cases, *title* contains the theme of the work presented in the paper. *Author's* information helps in identifying previous papers of the authors in the database. *Abstract* summarizes the paper and almost contains the important terms and theme of the paper. *Keywords* are the most weighted terms in assigning a document. This part mostly shows the domain of the paper. *References* can help in finding the cited paper's category. In case of the most papers category of *references*, it is highly **likely** to assign the paper to that category [28]. Among these features, we selected *title* and *keywords*. The selection of these is discussed in detail in the discussion section. The relevant information required for document classification is depicted in the Fig. 2.

The information about a document is first pre-processed. Pre-processing prepares the data for accurate classification. First step in the pre-processing is the removal of unnecessary words from the TD. From this relevant feature (*Title* and *Keywords*) *stopwords* are removed using a list containing 548 *stopwords* [29]. After removing the *stopwords,* these features are stemmed using the well-known porter stemmer algorithm [23]. Based on the pre-processed data, we apply our classification algorithm.

5. PROPOSED TECHNIQUE FOR SCIENTIFIC DOCUMENT CLASSIFICATION

Document classification process is depicted in the proposed framework (Fig. 3). Initially, the dataset is populated with the similar approach.. *Title* and *keywords* are extracted from each of the documents in the training dataset. Extracted *title* and keywords are pre-processed using *stop words removal* and *stemmer* module in the framework. Representation of the dataset with respect to the documents is given in Fig. 2 and Fig. 3.

In Fig. 3, the user issues a TD for category identification. The TD is parsed in the *dataextractor* module, in which relevant data (*Title* and *Keywords*) are retrieved. The data extracted is passed on to the *stopwordremoval* module, which removes the unnecessary words using the list provided in [29]. The remaining text is passed on to the *stemmer* module, which returns the stemmed result to the *matcher* module. For stemming, we used the well-known porter stemmer algorithm [23]. The *matcher* module then predicts the category of the TD using the existing dataset. The category result is returned to the user and dataset is updated with the relevant category/ies information of the TD.

Matcher module is the main module of the

proposed framework. The *matcher* classifies an input TD into their relevant category/ies. Proposed solution towards automated category identification is inspired from the well know evolutionary particle swarm optimization. Proposed solution overcomes the two well-known problems (Multi-class and Multi-level) in the solution towards automated category identification.

Initially, the TD is matched with all the documents in each category. Average similarity of each category is computed, among which highly similar (using Eq. 3) categories are selected. At each level, besides identifying the category, we find similarity among the similarity score of each category. The search space is reduced at the second level by assigning the TD to selected categories at first level. Our algorithm recursively reaches to the bottom level to assign the paper to its correct category.

Proposed solution towards the classification of new document is a recursive approach, as depicted in Fig. 4. Initially, the *TD* will be checked with the same number of documents from each category. Global best *gBest* among all the local best *pBest* available categories will be selected. In our case, more than one *gBest* can be selected based on the Eq. 3 among average similarity measures with each category. If the difference is higher than a certain threshold then more than one category can be selected for the new document. After selection at first level, the new document will be matched with all the subcategories of the selected category/ies. The process can be continued till the leaf level of the ACM CCS is achieved or the fitness function is achieved.

Formally, the solution can be formulated as:

$$C = \{\mathrm{A}, \mathrm{B}, \dots\dots, \mathrm{K}\}$$

where *C* is the set of categories

Each category contains sub-categories. For example

$$A = \{\mathrm{A}_0, A_1, \dots\dots, \mathrm{A}_m\}$$

Each sub-category may itself contain third level sub-categories. Each category contains a set of documents as for example

$$A = \{d_{a1}, d_{a2}, \dots\dots, \mathrm{d}_{am}\}$$
$$B = \{d_{b1}, d_{b2}, \dots\dots, \mathrm{d}_{bm}\}$$
$$\vdots$$
$$K = \{d_{k1}, d_{k2}, \dots\dots, \mathrm{d}_{km}\}$$

Each document contains a set of words among which $t_1, t_2, \dots\dots t_k$ are terms belonging to *title* and *keywords*, as a feature vector, as

$$D_{ci} = \{\mathrm{t}_1, t_3, \dots\dots, \mathrm{t}_k\}$$

Similarly, the new document is to classify contains terms of *title* and *keywords* as

$$\mathrm{TD} = \{\mathrm{t}_1, t_3, \dots\dots, \mathrm{t}_k\}$$

Membership of the TD (similarity) with each category is calculated as

$$\mu_{c_i}(\mathrm{TD}) = \frac{AVG(\sum_{i=1}^{\mathrm{n}} \mathrm{similarity}(TD, \mathrm{d}_{Ci}))}{\mathrm{n}}$$
$$= x_{ci} \ where \ n \ the \ minimum \ number \ of$$

$$documents \ in \ Ci \ \ Eq{:}2$$

For multi-topic classification, Eq. 3 is used. In Eq. 3, $max(x_{ci})$ is the maximum average similarity selected at any level in the hierarchy; whereas x_{ci} is the membership of the document in each category using Eq. 2, Ψ is the threshold defined by domain expert which can be the maximum similarity difference between any two categories. We used Levenshtein distance [30] as a similarity measure. Classification at the next level (lower level) will be performed only for the categories selected using equation 3. The document movement in the taxonomy is identified using the Eq 3.

$$D_t \in C_i \text{ where difference } (\max(x_{ci}) - x_{ci}) > \psi \ \dots . Eq{:}3$$

Concept of social interaction is applied in using the PSO. Each particle (category) takes part in classification of the TD category identification in the taxonomy. Position and velocity of TD using PSO is given by the Eq. 2 and Eq. 3. In our velocity equation, we are not using cognitive and

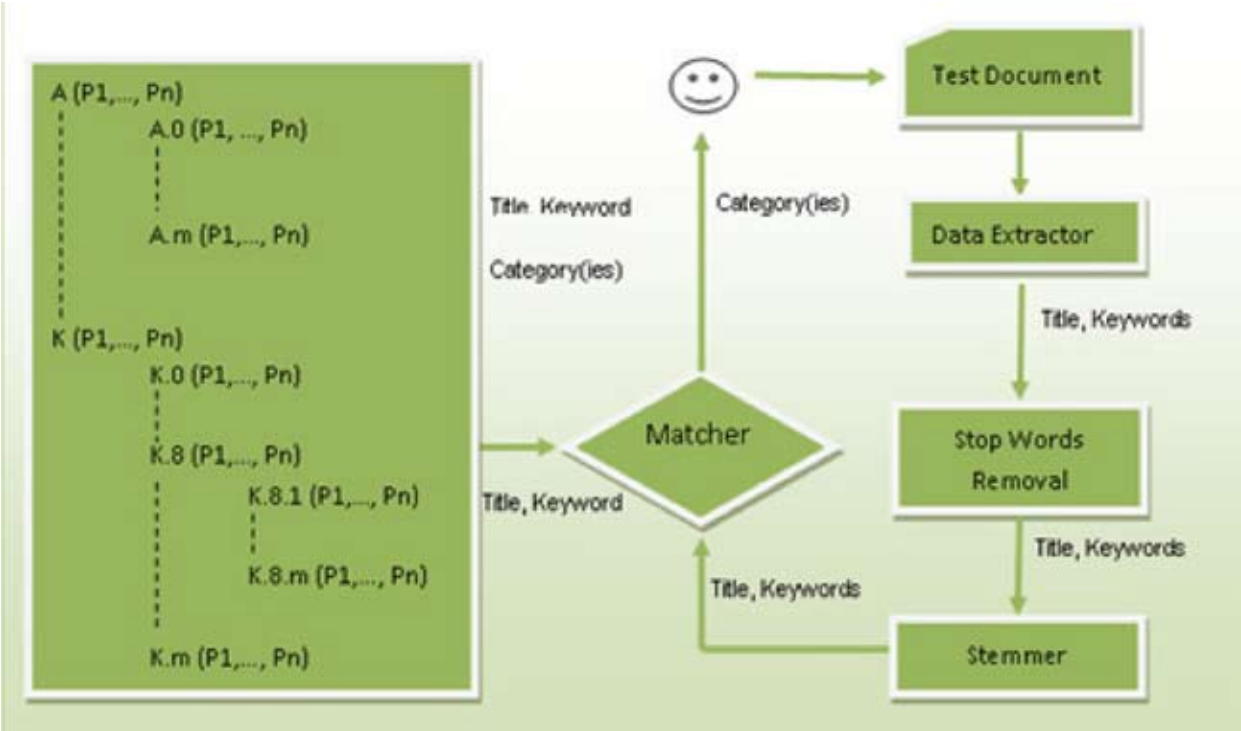

Fig. 3. Framework for classification of test document.

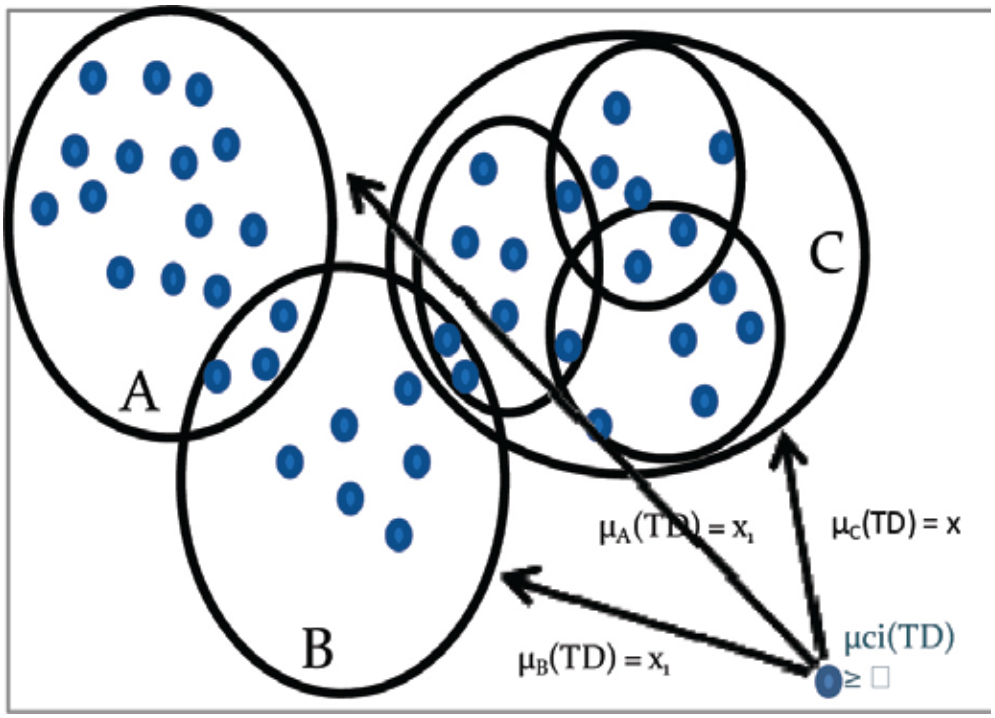

Fig. 4. Classification of test document.

social parameters of PSO, as scientific document classification is hierarchical in nature. And at each level, the document is classified to a category or some of the categories. At lower level, PSO is again applied on the selected categories using Eq. 3.

Fig. 5 explains the proposed algorithm inspired from PSO. This algorithm works as a central module (Matcher) of the framework shown in Fig. 4.Initially the category with minimum numbers of documents is selected from the available categories are selected. *pBest* and *gBest* for each category are calculated using Eq. 2 and Eq. 3 respectively. The resultant category/ies information of the TD is updated in the database.

6. EXPERIMENTAL RESULTS

We have implemented the proposed scheme on the Journal of Universal Computer Science (J.UCS) dataset. Our proposed dataset contains 1460 research publication. We have taken 2/3 documents for training dataset and 1/3 for test dataset. J.UCS has extended the ACM CCS with two more categories with *L* and *M*. Test accuracy result for 30 test documents from each category are shown in Fig. 7. The test sample selected was random in each category as shown in Fig. 7; each column shows the selected input documents in each category. The result of our test documents with each category is given in Fig. 6. The blue bar shows the correctly classified categories whereas the red bar shows the number of incorrect classified documents in each category. Overall accuracy of our approach is more than 84.71%. We have implemented our approach using MySQL as database with PHP.

We performed a set of experiment for the automation of topic identification in ACM CCS. In our first experiment, we used the features provided in the ACM CCS. For top level, we aggregated child features for a parent category. At each level, we stored all of the decedent's features for each category. We used this database for the classification of TD. We matched the extracted TD features with our stored features for each category in the database. After several tests, the classification was not accurate. We changed the similarity measure used for finding the relatedness between the TD features with the features for each category, but the results were not promising. After manual expectation of the extracted features, similarity with the stored features in the database we conclude that the features provided in the ACM CCS are not suitable for the automation of topic identification

Our second experiment was the selection of relevant information from test document to find similarity with individual documents in each category. Initially, we selected *title*, *keywords* and *abstract*, after a set of experiments we concluded that abstract diversify our classification results. When we tested the similar approach by excluding the *abstract*, the results were satisfying. *Abstract* contains a lot of text, which diversify the classification results. Another reason to exclude the *abstract* from text classification is the similarity measure which we used for our experimentations. After analyzing the result from our second experimentation for each individual category we observed that the categories having larger number of documents as compared to other categories, their results were comparably poor with the categories having less number of documents. The classification of a category having less number of documents was remarkable.

In our third set of experimentation, we selected the same number of document from each category. This time the classification for each category was relevant, closer to the results with other categories. Detail result for each category is shown in Fig. 7. The instance (*x y --- value*) in a cell represents *x* for original category, *y* for the system identified category and *value* representing the maximum average similarity. In some categories (*D, G, and K*) the misclassification error is relatively high as compared to the remaining categories.

One major problem in this experimentation is the error rate in training dataset. As previously used techniques for category identification were manual, in some of the cases in training documents we noticed that the assigned categories

```
For each category
      Initialize each category with minimum number of documents in a category among all
      the categories C
End

Do
          For each category
             Calculate similarity (Levenshtein distance) of TD with each document
              calculate the average similarity for each category as new pBest using Eq. 2
  End

Choose the category with the best average similarity value as gBest using Eq. 3

        For each category
           Calculate TD velocity (relevant categories for next level) using Eq3
           Update category information of the new TD in database
        End
While leaf level is not attained
```

Fig. 5. Classification of new document algorithm.

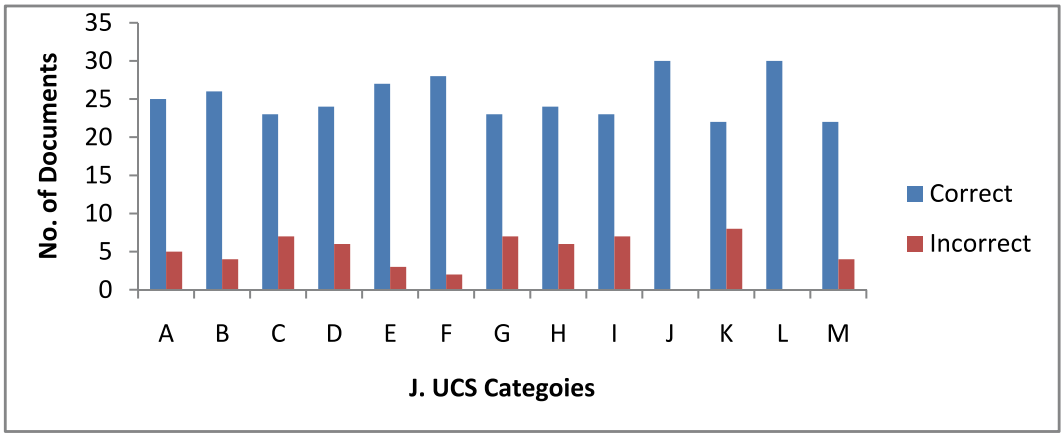

Fig. 6. Result of test documents classification with each category.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	A A---30.99	B M---23.78	C C---23.69	D D---30.01	E D---29.02	F D---29.23	G F---30.21	H H---33.32	I I---38.7	J J---29.24	K D---25.62	L L---24.66	M A---27.64
2	A A---33.19	B B---23	C C---23.48	D C---23.69	E G---25.31	F F---27.41	G G---26.72	H H---35.32	I I---24.66	J J---34.04	K K---24.63	L L---30.48	M H---25.93
3	A A---30.56	B B---22.13	C C---25.54	D D---29.02	E E---24.4	F F---28.36	G G---27.67	H H---35.43	I I---28.06	J J---25.04	K K---30.35	L L---33.44	M L---26.25
4	A A---30.21	B B---24.83	C C---25.7	D D---29.02	E E---22.78	F F---26.67	G G---24.05	H H---35.43	I I---29.2	J J---24.84	K K---23.05	L L---26.64	M L---30.48
5	A A---27.46	B B---22.36	C C---24.15	D D---30.01	E E---22.6	F F---30.81	G F---30.42	H H---33.32	I I---38.7	J J---27.59	K G---24.61	L L---24.91	M M---25.27
6	A A---30.35	B B---24.83	C C---25.54	D D---29.02	E E---24.32	F F---27.74	G G---24.49	H H---35.43	I I---29.2	J J---25.77	K K---30.35	L L---39.59	M M---25.61
7	A A---31.21	B B---25.94	C G---24.61	D D---23.19	E A---29.2	F F---24.64	G G---28.25	H H---28.67	I I---38.7	J J---33.19	K K---28.86	L L---31.78	M M---25.68
8	A A---29.2	B B---24.7	C C---28.13	D D---24	E E---24.07	F F---27.4	G J---25.34	H H---38.6	I I---38.7	J J---27.41	K K---26.21	L L---30.7	M M---27.2
9	A A---33.19	B B---24.23	C C---28.13	D C---24.15	E G---24.01	F F---30.2	G G---24.96	H H---35.32	I I---24.66	J J---34.04	K K---24.63	L L---30.48	M M---27.78
10	A J---34.7	B B---23.91	C C---25.25	D G---24.61	E A---22.92	F F---30.42	G G---27.55	H I---38.7	I F---25.26	J J---25.43	K K---30.72	L L---39.59	M M---27.89
11	A A---32.92	B B---24.52	C A---27.61	D D---27.41	E A---29.45	F F---30.21	G G---27.55	H I---38.7	I C---24.15	J J---34.7	K K---30.72	L L---39.59	M M---27.89
12	A A---29.2	B B---25.94	C D---27.41	D D---21.62	E E---25.12	F F---25.17	G G---26.15	H H---38.6	I I---38.7	J J---22.47	K K---28.86	L L---31.78	M M---28
13	A A---32.5	B J---24.67	C C---24.01	D D---26.02	E E---22.48	F B---24.67	G F---30.2	H H---33.32	I C---30.11	J J---27.56	K C---23.69	L L---22.08	M M---28.37
14	A A---29.89	B B---22.89	C D---24	D D---30.01	E E---22.41	F F---30.81	G F---27.41	H H---33.32	I I---38.7	J J---29.11	K I---26.32	L L---26.64	M M---28.37
15	A A---32.5	B C---25.25	C D---26.02	D F---26.71	E I---28.06	F F---25.26	G F---25.17	H H---36.32	I C---30.11	J J---25.9	K D---24	L L---23.59	M M---28.88
16	A A---23.76	B B---27.27	C C---30.11	D D---25.03	E E---23.66	F F---26.71	G I---26.46	H H---36.32	I H---38.6	J J---38.6	K K---23.26	L L---26.93	M M---28.88
17	A H---28.67	B B---24.67	C C---30.11	D H---28.67	E E---28.03	F F---24.69	G I---26.76	H H---36.32	I H---38.6	J J---33.32	K H---28.67	L L---26.25	M M---29.42
18	A H---39.69	B B---27.27	C C---30.11	D I---38.7	E E---28.03	F F---26.24	G G---24.61	H H---36.32	I H---38.6	J J---28.67	K K---23.26	L L---26.93	M M---29.42
19	A A---27.98	B B---22.23	C C---27.13	D F---30.81	E E---24.5	F F---28.36	G G---27.67	H H---33.13	I I---28.06	J J---27.98	K C---25.08	L L---23.32	M M---29.7
20	A A---32.4	B B---21.29	C C---27.13	D F---30.81	E E---24.7	F F---25.3	G J---25.35	H H---39.69	I I---26.76	J J---28.64	K K---29.11	L L---33.44	M M---29.7

Fig. 7. Detail result of random selected document in each category along with their results.

to documents are not relevant, while in some cases it was observed that a document was assigned to some extra categories, beside their main categories. Result of proposed technique can be improved by removing the error rate from the training dataset.

Our experimental results are better than the Bayesian approach presented in [13] with 83.75% classification accuracy tested on four categories. Similarly, the proposed approach accuracy 84.71% is better than Bayesian network learned from data with 76.25% accuracy and naïve Bayesian classifier with 82.5% accuracy respectively. Majority best evidence, majority Genetic programming approach and SVM results are compared with the proposed approach. Majority best evidence, Majority GP and SVM [26, 31] having performance accuracy of 53.60%, 57.74% and 57.74%respectively. The compared results with the approaches for scientific publication classification are given in Table 1. In our experiments, we have included all of the categories provided in J.UCS and provided results for each category. The other advantage of the proposed approach is to overcome the multi-class and multi-level classification of scientific publication to the taxonomy.

Table 1. Comparison of different approaches.

Approaches	Number of Categories	Average Accuracy
Bayesian Approach	4	83.75%
Bayesian Network learned from Data	4	76.25%
Naïve Bayesian	4	82.50%
Majority Best Evidence	11	53.60%
Majority GP	11	60.81%
SVM	11	57.74%
Proposed Approach	13	84.71.%

7. SUMMARY

Classification is, in general, a central problem in different domains. The multi-class classification is different to the ordinary classification problem. Similarly, classifying the document at different levels is also an important issue to many classification problems. Therefore, in this paper we have proposed a solution for both multi-class classification and multi-level classification. Our classification technique is an enhanced from of PSO in the context of document classification. Efficiency is achieved by reducing the size of features set and by considering the minimum number of documents in all categories. This solution can be applied for different domain where an instance can belong to multiple categories along with multi-level classification. We have implemented and tested our technique which provides better results as compared to existing techniques. This work will help authors in selecting the correct category for papers. Correct classification can, in terms, be quite useful for document retrieval and analysis.

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Research Article

Evaluation of Relative Efficacy of Hypertonic Saline and Lactated Ringer's Solutions in Experimentally-induced Hypo-volumic Shock in Dogs

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Abstract: This study was conducted to evaluate the relative efficacy of bolus administration of lactated Ringer's (LRS) and hypertonic saline (HSS) solutions at the same dose rate in hypovolaemic shock (HS). Twelve healthy adult Mongrel dogs were acclimatized and baseline data were obtained. The animals were then subjected to HS through shedding blood and mean arterial pressure (MAP) of ≤ 40 mm of Hg was considered as cut-off point for the development of HS. After 30 minutes of HS, the dogs were randomly divided into two equal groups (A & B) before initiation of treatment. The dogs of group A were infused with LRS @ 4 mLkg^{-1} body weight (BW) followed by slow administration of LRS @ 10 mL kg^{-1} BW, while animals of group B were administered with HSS @ 4 mL kg^{-1} BW followed by slow administration of normal saline @ 10 mL kg^{-1} BW. Relative efficacy was measured through MAP, haematological parameters, serum electrolytes and blood pH. All parameters were measured during shock, and 30, 60 minutes, 3, 6, 12, and 24 hours of post-infusion. The animals of group B showed better recovery from shock, while 50% mortality was observed in the group A animals at the monitoring time. In group B, the mean ±SD of MAP was 116±8.0, equivalent to baseline within 60 minutes of infusion but in group A, it was far less than the baseline value. Sodium elevated above baseline in the group B animals but within observing time it became close to the baseline. Blood pH was restored towards normal more rapidly in group B animals which was significantly different from the group A animals ($P < 0.05$). Recovery towards normal baseline values of measured variables was more rapid and sustained for the HSS treated group. It was concluded that HSS has a rapid effect in resuscitating the dogs from HS than LRS.

Keywords: Resuscitation, hypovolaemia, 7.5% NaCl solution, dogs

1. INTRODUCTION

Hypovolaemia is the principal cause of death in many of the traumatized dogs and diarrheic calves etc. Survival from hypovolaemic shock is dependent upon the restoration of cardiac out, oxygen transport and blood volume. So, for rapid recompensation, isotonic fluids have been widely used and considered key to success in rectification of this alarming condition [1].

Sodium is expected to elevate extracellular volume more than the transfused amount because hypertonic Na^+ solution attracts water out of the cells, thus expands plasma volume and increases the venous preload [2, 3]. It has been a matter of debate about the most suitable solution for the resuscitation of the animals from hypovolaemic shock and in recent years, resuscitation with hypertonic saline solution (HSS) has gained widespread acceptance because hypertonic salt is a potent volume expander. Hypertonic saline solution induces a rapid plasma volume, increase cardiac output, mean arterial pressure (MAP) and may improve survival [4, 5]. So, HSS seems likely to produce a more rapid resuscitation and beneficial effects than isotonic solutions.

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In this study, relative resuscitative efficacy of hypertonic saline solution and lactated Ringer's solution (LRS) was evaluated in experimentally induced hypovolaemic shock in the dogs at the same dose and infusion rates.

2. MATERIALS AND METHODS

Twelve adults (1±0.15 years old), healthy mongrel dogs of either sex were used in the study. During the acclimatization period of one week, all the animals were fed on the same feed and fresh water was available a*d libitum*. Blood, faecal and urine examination during this period indicated that these dogs were healthy.

Each dog was sedated with intramuscular administering Acepromazine + Ketamine hydrochloride @ 0.1 mg/kg + 4 mg/kg, respectively, after the subcutaneous administration of atropine sulphate (0.05 mg/kg of body weight). Medial side of the right limb was shaved for exposure of femoral artery and was cannulated with 22 gauge (Vasocan Brannula®) IV cannula for measurement of mean arterial blood pressure (MAP) by using saline sphygmomanometer. For infusion of solutions, cannula (Vasocan Brannula®) of 18 gauge was used percutaneous into the left cephalic vein for administration of allotted solutions. Dogs were bled through jugular vein for the induction of hypovolaemic shock and blood was collected in blood bags @ 40±5 ml/kg BW [6]. For the assessment of shock state, mean arterial pressure of 50 mm Hg was considered cut-off point to stop bleeding [6].

After 30 minutes of inducing shock, all animals were randomized into two groups (n=6). Group A was infused with lactated Ringer's solution @ 4 ml/kg, BW, and was maintained with the same solution @ 10 ml/kg, BW. Animals of group B were infused with hypertonic saline (7.5% sodium chloride) solution @ 4 ml/kg, BW, and maintained with isotonic saline solution (ISS) @ 10 ml/kg, BW. Both lactated Ringer's and hypertonic saline solutions were administered at the infusion rate of 20 ml/min.

Blood samples of all the animals were collected at baseline (before shock), during shock, 30, 60 minutes, 3, 6, 12, and 24 hours post infusion of respective solutions. The venous blood samples with anticoagulant were used to determine haemoglobin concentration (Hb conc.), haematocrit (Hct) and pH values, while the samples without anticoagulant were used to harvest serum to determine serum electrolytes (Na^+, K^+ and Cl^-) and bicarbonates (HCO_3^-). The Hb concentration was determined by cyannmethemoglobin method and Hct values were determined with microhematocrit method as described by Benjamin [7]. Serum Na^+, K^+ and Cl^- were determined with the help of EasyLyte Analyzers (Medica Corporation, Bedford) and HCO_3^- with the help of Microlab-200 (Vital Scientific, Merck, Netherlands).

The data are presented as mean ± SD (Table1). Statistical analyses were performed using t-test. All analysis was performed using the Statistical Software Package (SPSS Version 11.5). Statistical significance was assigned at $P < 0.05$.

3. RESULTS

Baseline values for the blood parameters of both the groups were not significantly different (Table 1). After induction of shock and infusion treatment 50% mortality was recorded within monitoring time in group A, while no mortality was observed in group B infused with hypertonic saline solution.

3.1 Haematological Parameters

After infusion of respective treatments, group B showed significant difference ($P < 0.05$) over group A by increasing their values from 3rd hour to onwards for Hb concentration and haematocrit (Table 1). After 24 hours, HSS recovered the haematocrit values near to baseline in animals of group B while LRS (group A) was unable to recover these values.

3.2. Mean Arterial Pressure

There were no differences regarding MAP between two groups before or during shock. However, MAP increased dramatically in group B after 30 minutes of treatment and this value was significantly higher ($P < 0.05$) than group A (Table 1). Mean arterial pressure in group B was near to baseline within 60 minutes. In group A, MAP did not recover even after 24 hours.

3.3. Serum Electrolytes and Biochemistry

There was no significant difference between the groups during shock in all the serum electrolytes

Table 1. The results of haematological parameters, mean arterial pressure, serum electrolytes and blood gases in both groups

				Time after treatment					
Variable	**Group**	**Baseline**	**Shock**	**30 min**	**60 min**	**3 hr**	**6 hr**	**12 hr**	**24 hr**
Number of Dogs	Group A	6	6	6	6	5	5	4	3
	Group B	6	6	6	6	6	6	6	6
Hct. (%)	Group A	37.0±3.7	14.7±0.5	14.7±0.7	15.5±0.5	16.3±0.5	17.3±0.6	19.2±0.8	20.0±0.6
	Group B	38.0±2.8	15.7±2.6	16.3±2.3	18.2±2.6	22.5±1.9*	24.7±2.3*	30.0±1.1*	32.7±2.0*
Hb conc. (g/dL)	Group A	14.3±2.3	8.9±1.8	9.0±1.8	9.2±1.9	9.4±2.0	9.8±1.6	10.2±1.6	10.7±1.5
	Group B	12.7±2.0	8.5±1.2	8.8±1.2	9.5±1.2	10.3±1.6*	10.8±1.1*	11.3±1.8*	11.9±1.9*
MAP (mm Hg)	Group A	126±10.6	48±4.0	57±6.1	65±4.9	79±5.1	88±4.9	90±6.8	90±6.8
	Group B	120±8.9	49±2.0	94±5.4*	116±8.0*	114±10.2*	117±13.3*	120±12.7*	118±9.8*
Sodium (mEq/L)	Group A	140±3.7	121±1.9	121±1.9	123±2.4	123±2.4	124±1.5	126±1.4	126±1.4
	Group B	141±3.4	120±4.2	135±4.1*	145±4.2*	153±4.8*	156±9.0*	147±6.1*	145±3.7*
Potassium (meq/L)	Group A	4.3±0.4	3.2±0.3	3.2±0.3	3.3±0.2	3.3±0.3	3.4±0.2	3.5±0.2	3.5±0.5
	Group B	4.3±0.3	3.5±0.2	3.7±0.2	3.8±0.3	3.9±0.3	4.1±0.4	4.1±0.3	4.1±0.3
Chloride (mEq/L)	Group A	119±4.7	109±5.4	109±5.3	109±5.3	107±6.7	111±5.1	112±5.3	112±5.4
	Group B	116±6.5	105±6.8	112±7.3	118±7.0	123±6.9*	122±3.9*	119±6.2	121±3.4*
HCO_3^- (meq/L)	Group A	19.5±2.6	13.5±1.5	12.8±0.7	13.6±1.3	14.1±1.1	15.3±1.2	15.0±0.8	15.8±0.9
	Group B	17.8±1.3	12.6±0.5	13.0±0.8	13.5±0.5	14.1±1.3	15.8±1.9	16.3±1.5	16.1±1.7
Blood pH	Group A	7.3±0.12	6.7±0.27	6.7±0.21	6.9±0.11	7.0±0.17	7.1±0.10	7.1±0.09	7.2±0.04
	Group B	7.3±0.20	7.1±0.52	7.0±0.55	7.2±0.44*	7.3±0.62	7.5±0.55*	7.4±0.31	7.3±0.21

*= Significant difference to other group; Group A= Lactated Ringer's solution; Group B= Hypertonic saline solution; Hb conc.= Haemoglobin concentration; Hct= Haematocrit; MAP= Mean Arterial pressure; HCO_3^- = Bicarbonates.

i.e. Na^+, K^+, & Cl^-. After treatment, sodium was near to baseline within 60 minutes in group B and then it increased to 156 ± 9.0 mEq/L which was much higher than the normal values and it was significantly higher ($P < 0.05$) than group A. Group A showed decreased level of sodium even after 24 hours. In case of potassium, there was no difference between both groups (Table 1).

After fluid resuscitation, there was no significant difference for bicarbonate (HCO_3^-) in two treatment groups. Blood pH was near to baseline within 3 h after administration of HSS to the animals in group B and it was significantly higher ($P < 0.05$) in this group as compare to group A. But after 6 h, there were no significant difference between both the groups (Table 1).

4. DISCUSSION

The objective of resuscitation of a hypovolaemic shocked patient is the restoration of the lost intravascular fluid volume [3]. The increase in fluid volume as well as accompanying increase in mean arterial pressure and oxygen transport has been assumed to contribute to an increased opportunity for treatment strategies. Haematocrit (Hct) and haemoglobin concentration indicate the presence of total cells including erythrocytes, leukocytes and platelets [8]. Any increase or decrease in values of both parameters showed the signs of dehydration and hypovolaemia, respectively. In the present study, these values decreased which clearly indicates reduction in the number of cells and ultimately plasma volume. Hypertonic saline solution generates plasma volume expansion rapidly by increasing Hct and Hb concentration values and also increased osmosis and promoted movement of endogenous fluid from the extravascular to intravascular space. While lactated Ringer's solution administration at low dose was unable to cause plasma volume

expansion.

It is widely accepted that HSS increases MAP more rapidly, principally, the rise in serum osmolality is thought to cause the major effect, by osmotic extraction of water from the interstitium and expansion of the intravascular volume [3]. In this study, MAP was rapidly restored in animals treated with HSS [5, 9, 10, 11]. The higher concentration of Na^+ ions is considered the main reason of MAP restoration because sodium is the main source which extracts water from intracellular space to the extracellular and increase serum osmolality.

Since sodium is largely confined to the extracellular compartments, HSS expands extracellular fluid space by extracting water from the cell. However, a rapid increase in serum sodium concentration could cause salt poisoning [2, 10]. The increase in serum sodium may also cause hypernatraemia, but at risk. Here, the administration of 4 ml/kg of HSS produced transient high sodium level (Table 1) but it remained under 160 mEq/L level which is the cut-off point of hypernatraemia [10] and no adverse effect was observed during the whole study and these results are inline with the results reported by other scientists [5, 9, 10, 11, 12]. On the other hand, LRS is hypotonic regarding sodium ions, so, sodium remained suppressed throughout the observing time. This low level of sodium didn't cause any transient change in extracting water from the cells.

Blood pH is an indicator of acidosis. During hypovolaemia, blood pH decreased which revealed that animals were suffering with acidemia. In our study, after HSS infusion, it increased satisfactory which showed that hypertonic saline could overcome acidosis as reported in previous studies [4, 12, 13]. Hence, HSS had shown well affectivity against acidosis.

5. CONCLUSIONS

Hypertonic saline solution (HSS), immediately followed by normal saline solution, seems to be a promising and practical method for the initial treatment of hypovolaemic shock. Use of Lactated Ringer's solution in similar situations will not help to resuscitate the hypovolaemic animals.

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Research Article

Comparative Phycoremediation of Sewage Water by Various Species of Algae

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Abstract: In this study sewage water treatment efficiency of *Chlorella vulgaris, Rhizoclonium hieroglyphicum* and mixed algae culture (*Microspora* sp., *Navicula* sp., *Lyngbya* sp., *Cladophora* sp., *Spirogyra* sp. and *Rhizoclonium* sp.) was compared. Sampled wastewater was analyzed for various parameters (i.e., COD, BOD, TS, TSS, TDS, TC, FC, TKN, TP, NO_3-N, PO_4, SO_4 and Cl^-) and concentrations of all these parameters in the untreated water were above the permissible limits of National Environmental Quality Standards of Pakistan (2000). Various algal species were used to treat sewage water by varying pond size, treatment duration, seasonal variation and growth rate of algae to arrive at the optimum outcome. Maximum percent reductions of various parameters, attained with *C. vulgaris,* were: chemical oxygen demand (98.3%), biochemical oxygen demand (98.7%), total Kjeldahl nitrogen (93.1%), total phosphorus (98.0%), nitrate (98.3%), phosphate (98.6%), chloride (94.2%), total coliforms (99.0%), faecal coliforms (99.0%) and total dissolved solids (98.2%) while maximum reduction in total suspended solids (92.0%) was obtained with a mixed algae culture and maximum increase in biomass by *R. hieroglyphicum* (0.75 g L^{-1} day^{-1}). Reduction in the concentration of pollutants in sewage water was to such a low level that it can be thrown in waterbodies without any further treatment.

Keywords: Algae, Chlorella, phycoremediation, pollution, pond, rhizoclonium, wastewater

1. INTRODUCTION

Phycoremediation is the use of micro- or macro-algae for the removal or biotransformation of pollutants, including nutrients and toxic chemicals from wastewater [1, 2]. The term phycoremediation was introduced by John [3] to refer to the remediation carried out by algae. Phycoremediation is comprised of several applications: (i) nutrient removal from municipal wastewater and effluents rich in organic matter; (ii) nutrient and xenobiotic compounds removal with the aid of algae-based biosorbents; (iii) treatment of acidic and metal wastewaters; (iv) CO_2 sequestration; (v) transformation and degradation of xenobiotics; and (vi) detection of toxic compounds with the aid of algae-based biosensors. A distinct comparison can be made between microalgae and other conventional technologies on the basis of nutrient removal [4].

The prevalent sewage treatment strategies are costly and not affordable in the developing countries of the world due to high consumption of energy. Sewage irrigation is another big problem which affects human health directly and indirectly. Pakistan is also facing problems related to contamination of water, where excessive and untreated sewage is discharged directly into water bodies as a result this water can not even be used for irrigation purposes. Sewage is also loaded with diseased pathogens, where they get their direct entry into the drinking water by leakage and cause serious intestinal infections especially in children. Prolong exposure to these hazards can cause several diseases including cancer and birth defects in young ones [5].

Microalgae can be used to treat municipal wastewater, industrial effluents and solid wastes aerobically as well as anaerobically. The use of algae to treat wastewater has been in practice for over 40

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years and first descriptions of this application was reported by Oswald [6]. So the use of microalgae for the treatment of municipal wastewater has been a subject of research and development for several decades. Extensive work has been conducted to explore the feasibility of using microalgae for wastewater treatment, especially for the removal of nitrogen and phosphorus from effluents [7, 8] which would otherwise result in eutrophication if dumped into water bodies.

Biological treatment enhances the removal of nutrients, heavy metals, pathogens and provides O_2 to heterotrophic aerobic bacteria for mineralization of organic pollutants, as a result CO_2 is released from bacterial respiration [9]. Photosynthetic aeration is important to reduce operation costs and limit the risks for pollutant volatilization under mechanical aeration. Recent studies have shown that microalgae can indeed support the aerobic degradation of various hazardous contaminants [10]. The hyper concentrated algal cultures, called 'activated algae' can be used to decrease the land and space requirements for wastewater treatment. This process removed nitrogen and phosphorus in a very short period of time, i.e., less than 1 hour [4]. Microalgae can be efficiently used to remove significant amount of nutrients because they require high amounts of nitrogen and phosphorus for the sysntheisis of proteins (45–60% of microalgae dry weight), nucleic acids and phospholipids. Nutrient removal can further be increased by NH_3 stripping or NH_3 precipitation due to the raise in pH [11]. This method is not appropriate for large scale wastewater treatment, therefore there is need to improve the technology.

The advantages of using algae for sewage treatment include low cost of the operation, possibility for assimilation of nitrogen and phosphorus into algal biomass, which consequently can be used as fertilizer thus avoiding sludge handling problem and the discharge of oxygenated effluent into the water body. In addition, the process has no carbon requirement for nitrogen and phosphorus removal, which is sustainable for the treatment of sewage effluents [2].

The current study was conducted to assess the pollution load carried by municipal drains into the water bodies and its treatment with environment friendly and naturally available fresh water algae. A comparison was made to find out the algal species with best reduction efficiency. Treatment efficiency was also measured in different months of the year to assess the effect of seasonal variations.

2. MATERIALS AND METHODS

2.1. Experimental Location and Layout

The experimental outdoor work was carried out in Botanic Garden and roof of Sustainable Development Study Centre, GC University, Lahore. The locations were selected in consideration of availability of enough light, space and ponds facility. The study was conducted to test the feasibility of growing algae as a sewage nutrients reduction agent. These experiments were carried out in batch mode and cultivation time for each condition was 8 days with three replications. The effect was studied in terms of algal growth and wastewater nutrients reduction. Aquatic cultures were conducted in synthetic ponds with different sized and having dimensions of 0.15 x 0.15 x 0.3 m (P1), 0.3 x 0.3 x 0.15 m (P2) and 0.9 x 0.15 x 0.45 m (P3) with final capacity 6.75 L, 13.5 L and 60.75 L respectively. In addition a pilot scale large pond with dimensions of 8 x 2 x 1 m (P4) and final capacity of 16000 L was used for sewage treatment [12]. Initially the algal biomass was kept constant in each pond with equal quantity of wastewater except in pond P4 where biomass was added on per litre bases deduced from other ponds.

2.2. Wastewater Sampling and Analysis

Waste water samples were collected from various sewage drains of Lahore city by grab sampling techniques; then these samples were mixed to form composit sample [13]. The composite sample was filtered to remove coarse particles and divided into three replicates. The analysis for various physicochemical parameters like biochemical oxygen demand (BOD), chemical oxygen demand (COD), total solids (TS), total dissolved solid (TDS), total suspended solids (TSS), total phosphorus (TP), total Kjeldhal nitrogen (TKN), Nitrate-nitorgen (NO_3-N), phosphate (PO_4) and chloride (Cl^-) was carried out by using standards methods

of American Public Health Association [14]. The sampled untreated wastewater was stored at 4°C until used for algae cultivation.

2.3. Algal Sampling, Identification and Incubation

R. hieroglyphicum was collected from the fish forms of the Department of Fisheries near Manawa police station, Lahore. *C. vulgaris* sample was obtainted from Mr. Tariq Rashid, working in the same department on utilization of *Chlorella species* as fish feed. Mixed algae culture was collected from Baradari in River Ravi, near Lahore and Botanic Garden of GC University, Lahore. These algal species were identified by adopting a standard methods described by Zarina et al [15, 16].

Collected algae were kept under optimum conditions as local outdoor cultures and inocula of algae were transferred in previously mentioned ponds for sewage treatment. All the experimental ponds with same quantity of selected algal species were kept under similar light intensity and photoperiod to get comparative results. Before analysis samples were filtered to remove algal biomass present in the sample after treatment, it was done to avoid the interference caused by algal biomass in final results. The rate of growth of these algal species was measured by the estimation of fresh weight. These algal species were harvested every second day to measure its fresh weight.

3. RESULTS AND DISCUSSION

3.1. Sewage Treatment with Algae

The collected sample was characterized for various parameters showing the results which were above National Environmental Quality Standards (NEQS) [17] values. It was observed from the study that in comparison to *R. hieroglyphicum* and mixed algae culture, the *C. vulgaris* was more efficient in treating wastewater (Table 1). Przytocka-Jusiak et al [18] reported that the nutrient reduction depends on the quantity of nutrients present in wastewater and extent of these nutrients absorbed by the algae for incorporation into algal tissues. *Chlorella* has the ability to uptake maximum phosphorous for the synthesis and accumulation of polyphosphates in their bodies.

3.2. Effect of Algal Species on Pollution Reduction

Various parameters of wastewater were analyzed after treatment with *R. hieroglyphicum*, mixed algae culture and *C. vulgaris*. Before analysis samples were filterd to remove algal biomass which can interfer with the results. The reduction percentage of COD (98.27%), BOD (98.69%), TKN (93.14%), TP (98%), NO_3-N (98.33%), PO_4 (98.63%), Cl^- (94.16%), TC (99%), FC (99%) and TDS (98.21%) was noted to be maximum with *C. vulgaris.* The treatemnt efficiency of *R. hieroglyphicum* was

Table 1. Characterization of untreated wastewater and quantity of pollutants reduced by *C. vulgaris*, *R. hieroglyphicum* and mixed algae culture.

Parameters	Untreated	Reduction with *C. vulgaris*	Reduction with *R. hieroglyphicum*	Reduction with Mixed algae culture	NEQS 2000
COD (mg/L)	721	708.5	705.8	683.6	150
BOD (mg/L)	407	401.7	396.4	389.8	80
TSS (mg/L)	970	836.1	869.6	891.9	200
TDS (mg/L)	4650	4567.2	4546.2	4546.2	3500
TC (MPN)	$1.6x\ 10^3$	$1.58x\ 10^3$	$1.58x\ 10^3$	$1.57x\ 10^3$	---
FC (MPN)	$1.6x\ 10^3$	$1.58x\ 10^3$	$1.58x\ 10^3$	$1.57x\ 10^3$	---
TKN (mg/L)	35	32.6	31.8	30.5	---
TP (mg/L)	22	21.6	21.1	20.9	---
NO_3-N (mg/L)	12	11.8	11.8	11.7	---
PO_4 (mg/L)	10	9.86	9.86	9.4	---
SO_4 (mg/L)	55	53.5	50.2	53.5	600
Cl^- (mg/L)	60	56.5	55.5	51.5	1000

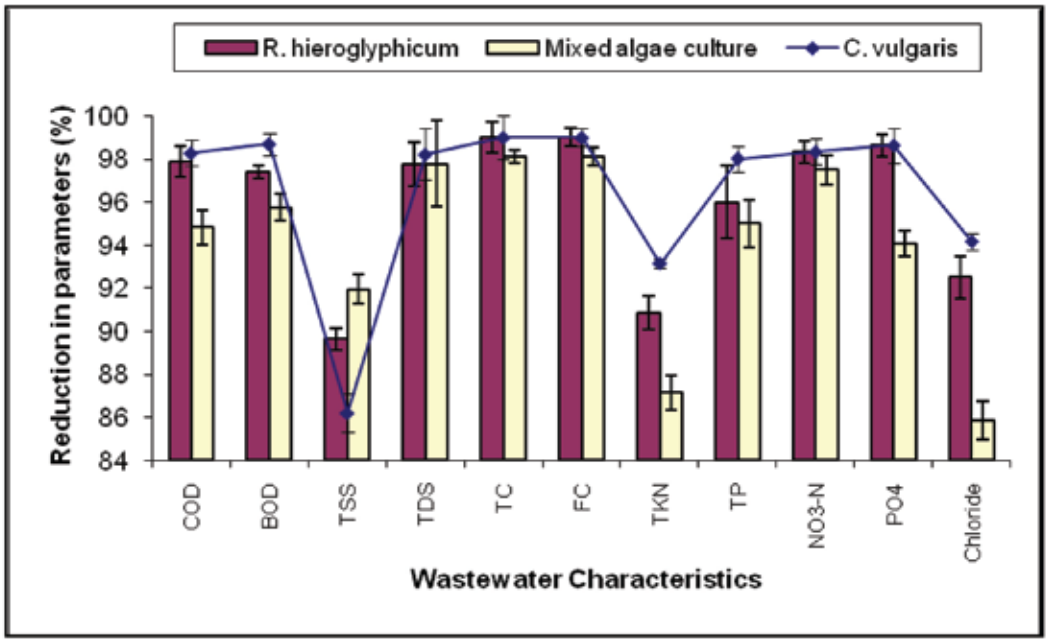

Fig. 1. Comparative efficiency of algal species in removing wastewater pollutants.

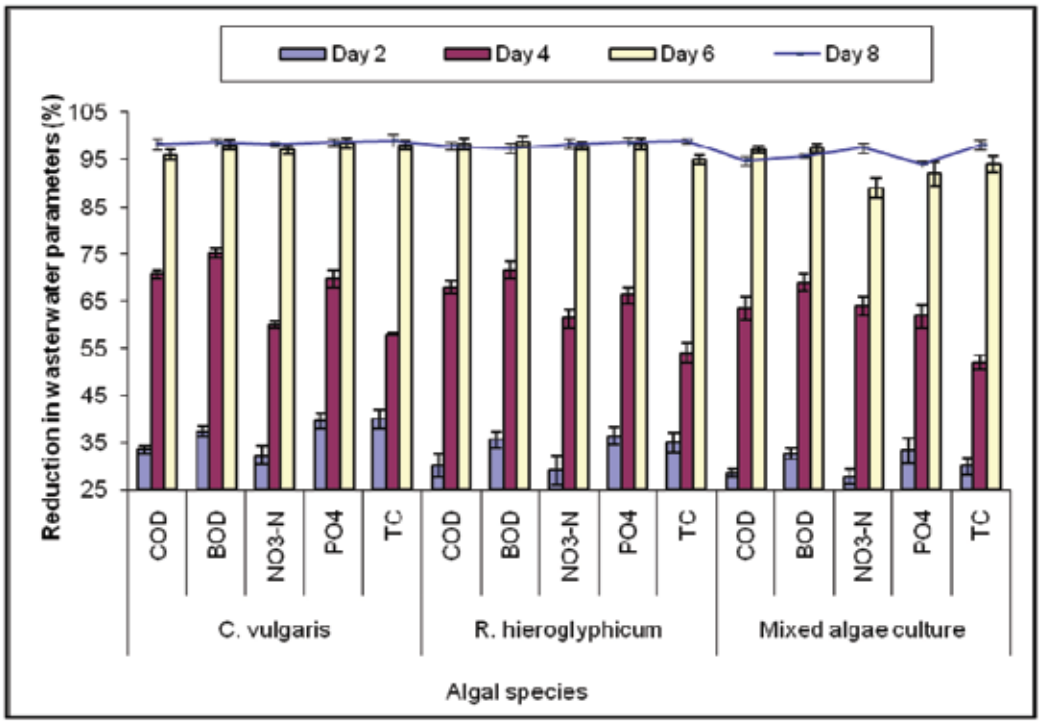

Fig. 2. Comparative percentage reduction of various parameters with different species of algae in time.

less than *C. vulgaris* but it was found to be higher than mixed algae culture. However the reduction percentage of TSS (91.3%) with mixed algae culture was higher than both *R. hieroglyphicum* and *C. vulgaris* (Figure 1). Microalgae showed efficient results in removing N and P from sewage-based wastewater either applied in free-swimming suspension or in an immobilized form. Increase in pH to 8 or above causes precipitation of phosphorous from the wastewater so it will not be wrong to say that nutrient reduction in wastewater occurs due to algal uptake as well as other reasons. Algal biomass contain phosphorous concentration ranging from 0.5-3.3 % of its dry mass which means 3-23 % of phosphorous was reduced due to algal uptake. In secondary treated wastewater more than 80% reduction of nitrate and total phosphorous was shown by various species of *Chlorella* and *Scenedesmus* [19, 20, 21].

3.3. Effect of Treatment Time on Pollution Reduction

Treatment duration has profound effect on treatment efficiency as the treatment time increases, algal biomass increases and it absorbs more nutrients from wastewater which results in the reduction of those nutrients. In the current study treatment time was taken in days (0-8) and wastewater was analyzed after every two days. It was observed that reduction became constant on 8th day with all the three types of algae however minimum reduction was achieved with mixed algae culture (Figure 2). Algae species *Chlorella* was widely applied for wastewater treatment and had proven abilities of removing nitrogen, phosphorus, and chemical oxygen demand (COD) with different retention times ranging from 10 hrs to 42 days [22].

3.4. Increase in Biomass of Algae in Wastewater

Increase in biomass of algae with the passage of time was measured by taking into account its fresh weight. Increase in biomass was observed from day 2 to 8 in all three types of algae. As five litre of wastewater was used to grow algal species for measurement of growth so increase in biomass on 8th day was divided by five for per litre and 8 for per day calculation. Maximum growth rate (0.75g.L^{-1}.day^{-1}) was shown by *R. hieroglyphicum*

Table 2. Increase in fresh weight of algal species with the passage of time in five litre of wastewater.

Algal species	Algal biomass (g)	Days					Increase in weight *(g. L^{-1} .day^{-1})
		0	2	4	6	8	
C. vulgaris	Fresh weight	100	102.7±2	115±0.6	121.8±2.3	122±1.5	0.55
R. hieroglyphicum	Fresh weight	100	109±1.1	120±2.1	128.5±1.6	130.01±1.8	0.75
Mixed algae culture	Fresh weight	100	105.5±0.8	118.4±0.9	122.3±1.5	124.0±1	0.6

* 8th day increase in biomass was divided by 5 (for L^{-1}) and 8 (for day^{-1}) measurement

Table 3. Percent reduction in various parameters with algal species in different sized ponds.

Parameters	*C. vulgaris*				*R. hieroglyphicum*				Mixed algae culture			
	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4
COD	89.4	98.6	88.1	98.7	86.6	98.8	89.9	98.5	85.9	94	87.2	98.7
BOD	93.7	98.8	92.9	98.9	89.5	98.9	93.7	98.8	90	93.3	92.2	98.9
TS	86.2	95.3	93.1	95.1	84.2	97.6	87.5	92.4	83.5	96.2	89.4	97.6
TC	94.1	99	95.2	99	92.4	99	93.8	98.2	93.2	98.8	96.1	99
TKN	89.4	97.7	87.7	97.2	86.3	97.7	90.9	95.8	85.4	95.3	90.2	97.7
TP	89.7	93.8	92.6	94	86.9	98.4	91	93.5	86.1	97	92.9	98.4
NO_3-N	91.2	98.1	94.7	98.5	89.1	98.9	92.8	95.9	89	94.5	92.1	98.9
PO_4	88.4	94.3	88.9	95	86	98.7	89.9	92.4	85.7	97.1	89.4	98.7

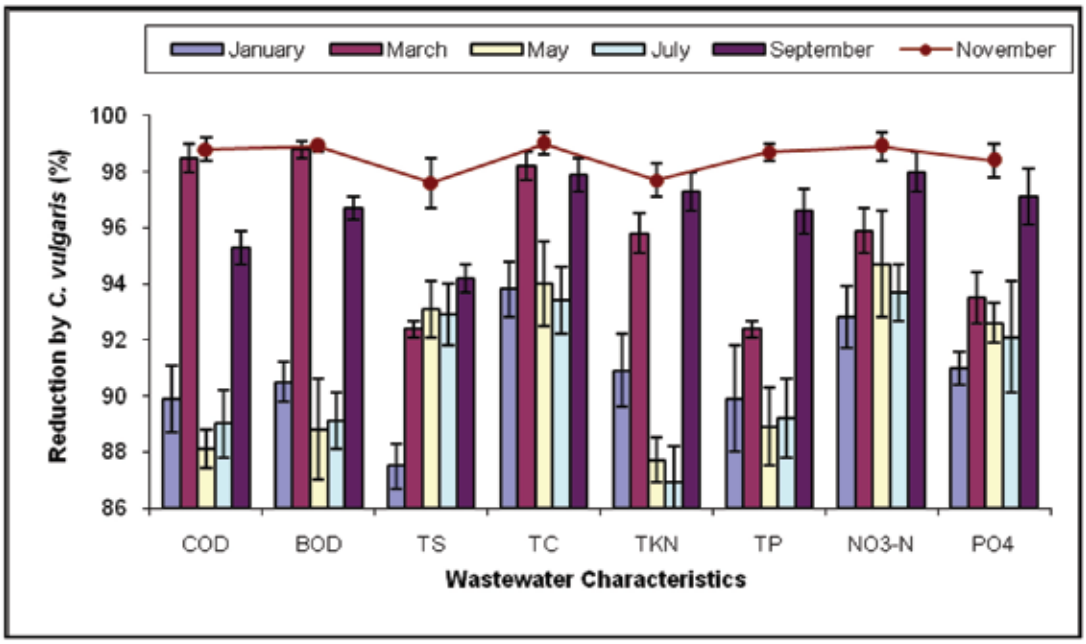

Fig. 3a. Percent reduction in wastewater characteristics in different months of the year with *C. vulgaris* species.

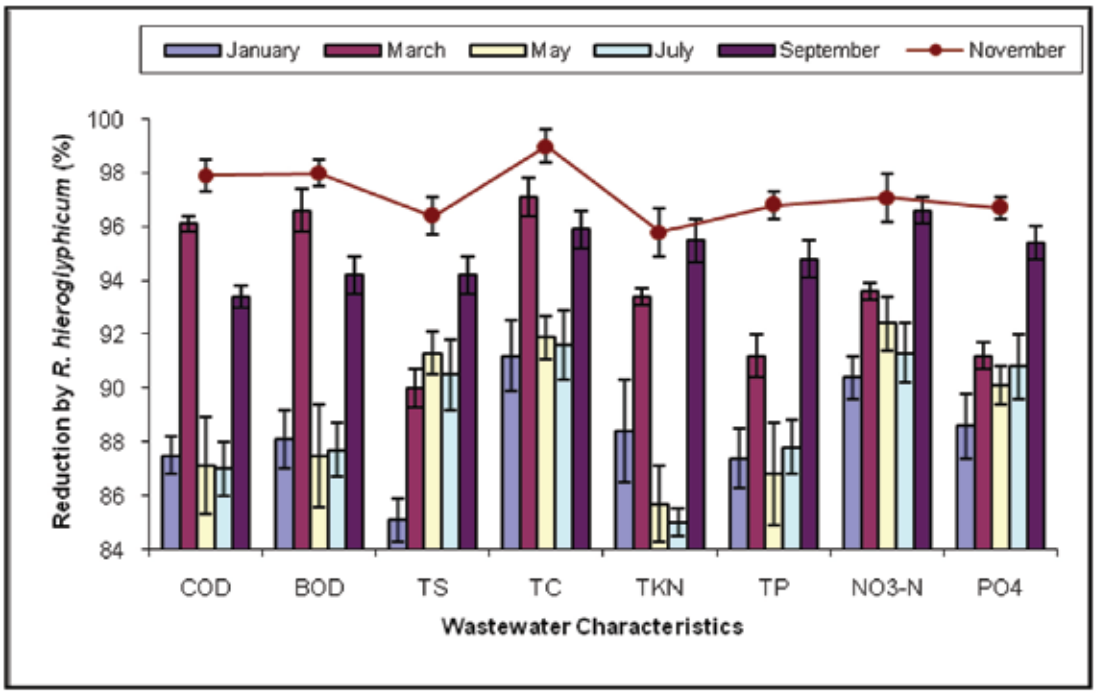

Fig. 3b. Percent reduction in wastewater characteristics in different months of the year with *R. hieroglyphicum*.

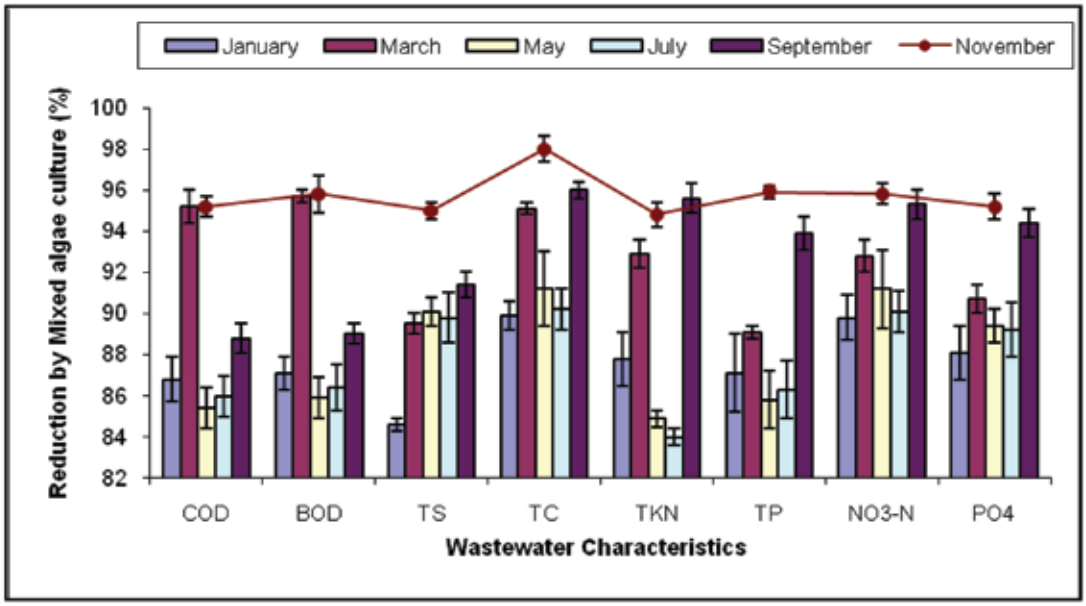

Fig. 3c. Percent reduction in wastewater characteristics in different months of the year with mixed algae culture.

while the increase in fresh weight resulted by *C. vulgaris* and mixed algae culture was observed to be 0.55 g L^{-1} day^{-1} and 0.6 g L^{-1} day^{-1}, respectively. It was also obseved that increase in growth became almost constant after 6th day due to deficiency in the availablity of the nutrients in the medium (Table 2). Ruiz-Marin et al [20] performed many experiment under batch culture condition in which microalgae showed high growth rates in initial days but growth and chlorophyll contents were decreased after four cycles of culture indicating collapse of the culture due to nutrient deficiency.

3.5. Effect of Pond Size in Pollution Reduction

Ponds of different shapes and sizes (P1, P2, P3 & P4) were selected to find out the relationship of exposed surface to pollution reduction. Results showed that maximum reduction was carried out in P2 and P4 with all three types of algae. It was because in these pond there was more surface area exposed for algae to be incontact with the wastewater and more light was available for the growth of the algal species. *C. vulgaris* also showed more reduction in P1 and P3 (Table 3) as compared to *R. hieroglyphicum* and mixed algae culture because microalgae remained suspended in the wastewater while macroalge come at the surface mostly therefore the wastewater at the depth was not incontact with the biomass which can be the reason of least reduction so in order to get maximum reduction a pond should be designed in such a way that it should have more surface area exposed to light and shallow water for maximum penetration of light.

3.6. Effect of Seasonal Variations on Pollution Reduction

In order to evaluate the seasonal variation on pollution reduction all three types of algal cultures were grown under optimum conditions in alternative months of the year. Experiments were conducted for 6 alternative months by renewal of sewage and algae but the initial concentration of pollutants in experimental wastewater was kept similar in all the selected months with same initial quantity of biomass of three selected algae. It was concluded from these experiments that maximum pollution reduction was observed in the month of november and march with all three types of algal cultures (Figure 3a,3b & 3c). It was due to moderate temperature found in these months which promote algal growth while minimum reduction was found in January, May and July due to low temperature in January and very high temperatur in May and July. It was observed that the reduction percentage of

pollutants in sewage in all the months was enough to be used for irrigation purposes. Green algae demands more nitrogen and phosphorous than do many other species, and they can take up generous nitrogen when the phosphorous content is relatively high [23].

4. CONCLUSIONS

The study revealed that algae are an effective organism for the reduction of biological and chemical pollutants from sewage water. Growth rate indicated that algae can luxuriously grow in wastewater medium due to availability of all necessary nutrients. Wastewater treated by algae can be used for irrigation purpose or released into water bodies as all the measuring parameters were in the permissible limits of National Environmental Quality Standards (2000) with selected species of algae. The study showed that phycoremediation process can be inversely affected by high temperature (May and July) and low temperature (January) but the reduction of pollutants in these months was enough to bring the remaining concentration into permissible limits. Consequently, this algal based treatment can be a sustainable technique for wastewater treatment.

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Research Article

Remarks on (1,2)*-αĝ-Homeomorphisms

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Abstract: The aim of this paper is to introduce two new class of functions called (1,2)*-αĝ-homeomorphisms and strongly (1,2)*-αĝ-homeomorphisms using (1,2)*-αĝ-closed sets and study their basic properties in bi-topological spaces.

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1. INTRODUCTION

Njastad [16] introduced α-open sets. Maki et al. [14] generalized the concepts of closed sets to α-generalized closed (briefly αg-closed) sets which are strictly weaker than α-closed sets. Veera Kumar [30] defined ĝ-closed sets in topological spaces. El Monsef et al. [1] introduced αĝ-closed sets which lie between α-closed sets and αg-closed sets in topological spaces.

Maki et al [15] introduced the notion of generalized homeomorphisms (briefly g-homeomorphism) which are generalizations of homeomorphisms in topological spaces. Subsequently, Devi et al [6] introduced two class of functions called generalized semi-homeomorphisms (briefly gs-homeomorphism) and semi-generalized homeomorphisms (briefly sg-homeomorphism). Quite recently, Zbigniew Duszynski [32] has introduced αĝ-homeomorphisms in topological spaces.

It is well-known that the above mentioned topological sets and functions have been generalized to bitopological settings due to the efforts of many modern topologists [see 7, 9, 10, 17-26]. In this present paper, we introduce two new class of bitopological functions called (1,2)*-αĝ-homeomorphisms and strongly (1,2)*-αĝ-homeomorphisms by using (1,2)*-αĝ-closed sets. Basic properties of these two functions are studied and the relation between these types and other existing ones are established.

2. PRELIMINARIES

Throughout this paper, (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) (briefly, X, Y and Z) will denote bitopological spaces.

2.1. Definition

Let S be a subset of a bitopological space X. Then S is said to be $\tau_{1,2}$-open [9] if $S = A \cup B$, where $A \in \tau_1$ and $B \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-

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closed.

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

2.2. Definition [9]

Let S be a subset of a bitopological space X. Then

(1) the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap$ {F : S $\subseteq$ F and F is $\tau_{1,2}$-closed}.

(2 the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup$ {F : F $\subseteq$ S and F is $\tau_{1,2}$-open}.

2.3 . Definition

A subset A of a bitopological space X is called

(1) (1,2)*-semi-open set [10] if A $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)).

(2) (1,2)*-α-open set [10] if A $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))).

(3) regular (1,2)*-open set [17] if A = $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)).

The complements of the above mentioned open sets are called their respective closed sets.

The (1,2)*-semi-closure (resp. (1,2)*-α-closure) of a subset A of a bitopological space X, denoted by (1,2)*-scl(A) (resp. (1,2)*-αcl(A)), is the intersection of all (1,2)*-semi-closed (resp. (1,2)*-α-closed) sets of X containing A.

2.4. Definition

A subset A of a bitopological space X is called

(1) (1,2)*-generalized closed (briefly, (1,2)*-g-closed) [19] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

(2) (1,2)*-semi-generalized closed (briefly, (1,2)*-sg-closed) [21] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X.

(3) (1,2)*-generalized semi-closed (briefly, (1,2)*-gs-closed) [22] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

(4) (1,2)*-ĝ-closed [7] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X.

(5) (1,2)*-αg-closed [18] if (1,2)*-αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

The complements of the above mentioned closed sets are called their respective open sets.

(6) (1,2)*-αĝ-closed [7] if (1,2)*-αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-ĝ-open in X.

2.5. Definition

A function f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is called (1,2)*-g-open [22] (resp. (1,2)*-ĝ-open [26], (1,2)*-open [20], (1,2)*-sg-open [22], (1,2)*-gs-open [22], (1,2)*-α-open [26], (1,2)*-αg-open [23], (1,2)*-αĝ-open [26]) if the image of every $\tau_{1,2}$-open set in X is (1,2)*-g-open (resp. (1,2)*-ĝ-open, $\sigma_{1,2}$-open, (1,2)*-sg-open, (1,2)*-gs-open, (1,2)*-α-open, (1,2)*-αg-open, (1,2)*-αĝ-open) in Y.

2.6. Definition

A function f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is called

(1) (1,2)*-g-continuous [21] if f^{-1}(V) is (1,2)*-g-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(2) (1,2)*-sg-continuous [21] if f^{-1}(V) is (1,2)*-sg-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(3) (1,2)*-gs-continuous [21] if f^{-1}(V) is (1,2)*-gs-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(4) (1,2)*-ĝ-continuous [23] if f^{-1}(V) is (1,2)*-ĝ-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(5) (1,2)*-continuous [17] if f^{-1}(V) is $\tau_{1,2}$-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

2.7. Definition [22]

A function f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is called

(1) (1,2)*-g-homeomorphism if f is bijection, (1,2)*-g-open and (1,2)*-g-continuous.

(2) (1,2)*-sg-homeomorphism if f is bijection, (1,2)*-sg-open and (1,2)*-sg-continuous.

(3) (1,2)*-gs-homeomorphism if f is bijection, (1,2)*-gs-open and (1,2)*-gs-continuous.

(4) (1,2)*-homeomorphism if f is bijection, (1,2)*-open and (1,2)*-continuous.

2.8. Definition [26]

A function f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is called

(1) (1,2)*-α-continuous if $f^{-1}(V)$ is (1,2)*-α-open in X, for every $\sigma_{1,2}$-open set V of Y.

(2) (1,2)*-αĝ-continuous if $f^{-1}(V)$ is (1,2)*-αĝ-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(3) (1,2)*-αĝ-irresolute if $f^{-1}(V)$ is (1,2)*-αĝ-closed in X, for every (1,2)*-αĝ-closed set V of Y.

2.9. Definition [25]

A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(1) pre-(1,2)*-α-closed (resp. pre (1,2)*-α-open) if the image of every (1,2)*-α-closed (resp. (1,2)*-α-open) in X is (1,2)*-α-closed (resp. (1,2)*-α-open) in Y.

(2) (1,2)*-α-irresolute if $f^{-1}(V)$ is (1,2)*-α-open in X, for every (1,2)*-α-open set V of Y.

(3) (1,2)*-gc-irresolute if $f^{-1}(V)$ is (1,2)*-g-closed in X, for every (1,2)*-g-closed set V of Y.

(4) (1,2)*-α-homeomorphism if f is bijection, (1,2)*-α-irresolute and pre-(1,2)*-α-closed.

2.10. Remark [7]

(1) Every (1,2)*-α-closed set is (1,2)*-αĝ-closed but not conversely.

(2) Every (1,2)*-αĝ-open set is (1,2)*-gs-open but not conversely.

3. (1,2)*-AĜ-HOMEOMORPHISMS

3.1. Definition

(1) A bijective function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called a strongly (1,2)*-αĝ-closed (resp. strongly (1,2)*-αĝ-open) if the image of every (1,2)*-αĝ-closed (resp. (1,2)*-αĝ-open) set in X is (1,2)*-αĝ-closed (resp. (1,2)*-αĝ-open) of Y.

(2) A bijective function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called an (1,2)*-αĝ-homeomorphism if f is both (1,2)*-αĝ-open and (1,2)*-αĝ-continuous.

3.2. Theorem

Every (1,2)*-homeomorphism is (1,2)*-αĝ-homeomorphism.

Proof

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be (1,2)*-homeomorphism. Then f is bijective, (1,2)*-open and (1,2)*-continuous function. Let U be an $\tau_{1,2}$-open set in X. Since f is (1,2)*-open function, f(U) is an $\sigma_{1,2}$-open set in Y. Every $\tau_{1,2}$-open set is (1,2)*-αĝ-open and hence f(U) is (1,2)*-αĝ-open in Y. This implies f is (1,2)*-αĝ-open. Let V be a $\sigma_{1,2}$-closed set in Y. Since f is (1,2)*-continuous, $f^{-1}(V)$ is $\tau_{1,2}$-closed in X. Thus $f^{-1}(V)$ is (1,2)*-αĝ-closed in X and therefore, f is (1,2)*-αĝ-continuous. Hence, f is an (1,2)*-αĝ-homeomorphism.

3.3. Remark

The converse of Theorem 3.2 need not be true as shown in the following example.

3.4. Example

Let X = {a, b, c}, τ_1 = {ϕ, X} and τ_2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b}}. Then the sets in {ϕ, Y, {a}, {b}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in Y. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Then f is a (1,2)*-αĝ-homeomorphism but f is not a (1,2)*-homeomorphism.

3.5. Proposition

For any bijective function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ the following statements are equivalent.

(1) $f^{-1} : (Y, \sigma_1, \sigma_2) \to (X, \tau_1, \tau_2)$ is (1,2)*-αĝ-continuous function.

(2) f is a (1,2)*-αĝ-open function.

(3) f is a (1,2)*-αĝ-closed function.

Proof

(1) ⇒ (2): Let U be an $\tau_{1,2}$-open set in X. Then X – U is $\tau_{1,2}$-closed in X. Since f^{-1} is (1,2)*-αĝ-continuous, $(f^{-1})^{-1}(X - U)$ is (1,2)*-αĝ-closed in Y. That is f(X – U) = Y – f(U) is (1,2)*-

αĝ-closed in Y. This implies f(U) is (1,2)*-αĝ-open in Y. Hence f is (1,2)*-αĝ-open function.

(2) ⇒ (3): Let F be a $\tau_{1,2}$-closed set in X. Then X – F is $\tau_{1,2}$-open in X. Since f is (1,2)*-αĝ-open, f(X–F) is (1,2)*-αĝ-open set in Y. That is Y – f(F) is (1,2)*- αĝ-open in Y. This implies that f(F) is (1,2)*-αĝ-closed in Y. Hence f is (1,2)*- αĝ-closed.

(3) ⇒ (1): Let V be a $\tau_{1,2}$-closed set in X. Since f is (1,2)*-αĝ-closed function, f(V) is (1,2)*-αĝ-closed in Y. That is $(f^{-1})^{-1}(V)$ is (1,2)*-αĝ-closed in Y. Hence f^{-1} is (1,2)*-αĝ-continuous.

3.6. Proposition

Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a bijective and (1,2)*-αĝ-continuous function. Then the following statements are equivalent:

(1) f is a (1,2)*-αĝ-open function.

(2) f is a (1,2)*-αĝ-homeomorphism.

(3) f is a (1,2)*-αĝ-closed function.

Proof

(1) ⇒ (2): Let f be a (1,2)*-αĝ-open function. By hypothesis, f is bijective and (1,2)*-αĝ-continuous. Hence f is a (1,2)*-αĝ-homeomorphism.

(2) ⇒ (3): Let f be a (1,2)*-αĝ-homeomorphism. Then f is (1,2)*-αĝ-open. By Proposition 3.5, f is (1,2)*-αĝ-closed function.

(3) ⇒ (1): It is obtained from Proposition 3.5.

3.7. Theorem

Every (1,2)*-α-homeomorphism is (1,2)*-αĝ-homeomorphism.

Proof

Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a (1,2)*-α-homeomorphism. Then f is bijective, (1,2)*-α-irresolute and pre-(1,2)*-α-closed. Let F be $\tau_{1,2}$-closed in X. Then F is (1,2)*-α-closed in X. Since f is pre-(1,2)*-α-closed, f(F) is (1,2)*-α-closed in Y. Every (1,2)*-α-closed set is (1,2)*-αĝ-closed and hence f(F) is (1,2)*-αĝ-closed in Y. This implies f is (1,2)*-αĝ-closed function. Let V be a $\sigma_{1,2}$-closed set of Y. Thus V is (1,2)*-α-closed in Y. Since f is (1,2)*-α-irresolute $f^{-1}(V)$ is (1,2)*-α-closed in X. Thus $f^{-1}(V)$ is (1,2)*-αĝ-closed in X. Therefore f is (1,2)*-αĝ-continuous. Hence f is a (1,2)*-αĝ-homeomorphism.

3.8. Remark

The following Example shows that the converse of Theorem 3.7 need not be true.

3.9. Example

Let X = {a, b, c}, τ_1 = {ϕ, X} and τ_2 = {ϕ, X, {a}}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in X. Moreover, the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called (1,2)*-α-closed in X and the sets in {ϕ, X, {b}, {c}, {b, c}} are called (1,2)*-α-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y} and σ_2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {a, b}} are called (1,2)*-α-closed in Y and the sets in {ϕ, Y, {c}} are called (1,2)*-α-open in Y. Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Then f is a (1,2)*-αĝ-homeomorphism but f is not a (1,2)*-α-homeomorphism.

3.10. Remark

Next Example shows that the composition of two (1,2)*-αĝ-homeomorphisms is not always a (1,2)*-αĝ-homeomorphism.

3.11. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in X. Let Y= {a, b, c}, σ_1 = {ϕ, Y} and σ_2 = {ϕ, Y, {a}}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c},

{a, b}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in Y. Let Z = {a, b, c}, η_1 = {ϕ, Z} and η_2 = {ϕ, Z, {a, b}}. Then the sets in {ϕ, Z, {a, b}} are called $\eta_{1,2}$-open and the sets in {ϕ, Z, {c}} are called $\eta_{1,2}$-closed. Also the sets in {ϕ, Z, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Z and the sets in {ϕ, Z, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in Z. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be two identity functions. Then both f and g are (1,2)*-αĝ-homeomorphisms. The set {a, c} is $\tau_{1,2}$-open in X, but (g o f)({a, c}) = {a, c} is not (1,2)*-αĝ-open in Z. This implies that g o f is not (1,2)*-αĝ-open and hence g o f is not (1,2)*-αĝ-homeomorphism.

3.12. Theorem

Every (1,2)*-αĝ-homeomorphism is (1,2)*-gs-homeomorphism but not conversely.

Proof

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a (1,2)*-αĝ-homeomorphism. Then f is a bijective, (1,2)*-αĝ-open and (1,2)*-αĝ-continuous function. Let U be an $\tau_{1,2}$-open set in X. Then f(U) is (1,2)*-αĝ-open in Y. Every (1,2)*-αĝ-open set is (1,2)*-gs-open and hence, f(U) is (1,2)*-gs-open in Y. This implies f is (1,2)*-gs-open function. Let V be $\sigma_{1,2}$-closed set in Y. Then $f^{-1}(V)$ is (1,2)*-αĝ-closed in X. Hence $f^{-1}(V)$ is (1,2)*-gs-closed in X. This implies f is (1,2)*-gs-continuous. Hence f is (1,2)*-gs-homeomorphism.

3.13. Remark

The following Example shows that the converse of Theorem 3.12 need not be true.

3.14. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in X. Moreover, the sets in {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-gs-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}} are called (1,2)*-gs-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Moreover, the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {a}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-g-closed and (1,2)*-g-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*-gs-homeomorphism but f is not a (1,2)*-αĝ-homeomorphism.

3.15. Remark

The following Examples show that the concepts of (1,2)*-αĝ-homeomorphisms and (1,2)*-g-homeomorphisms are independent of each other.

3.16. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {a, b}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}, {c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-g-closed in X. Moreover, the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called (1,2)*-αĝ-open and (1,2)*-ĝ-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {b}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}, {a, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {a}, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}, {b, c}} are called (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-g-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-g-open in Y. Define a function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = b, f(b) = a and f(c) = c. Then f is a (1,2)*-αĝ-homeomorphism but f is not a (1,2)*-g-homeomorphism.

3.17. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-g-closed in X. Moreover, the sets in {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open and (1,2)*-g-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Then the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {a}, {b, c}} are called (1,2)*-αĝ-closed

and (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-gs-closed and (1,2)*-gs-open in Y. Define a function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = b, f(b) = c, f(c) = a. Then f is a (1,2)*-g-homeomorphism but f is not a (1,2)*- αĝ-homeomorphism.

3.18. Remark

(1,2)*-αĝ-homeomorphisms and (1,2)*-sg-homeomorphisms are independent of each other as shown below.

3.19. Example

The function f defined in Example 3.16 is (1,2)*- αĝ-homeomorphism but not (1,2)*-sg-homeomorphism.

3.20. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}$-open and (1,2)*-αĝ-open in X; the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}$-closed and (1,2)*-αĝ-closed in X. Also, the sets in {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-sg-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}} are called (1,2)*-sg-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Then the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {a}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-sg-closed and (1,2)*-sg-open in Y. Define a function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = b, f(b) = a and f(c) = c. Then f is (1,2)*-sg-homeomorphism but not (1,2)*-αĝ-homeomorphism.

4. STRONGLY (1,2)*-AĜ-HOMEOMORPHISMS

4.1. Definition

A bijection f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is said to be strongly (1,2)*-αĝ-homeomorphism if f is (1,2)*-αĝ-irresolute and its inverse f^{-1} is also (1,2)*-αĝ-irresolute.

4.2. Theorem

Every strongly (1,2)*-αĝ-homeomorphism is (1,2)*-αĝ-homeomorphism.

Proof

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be strongly (1,2)*-αĝ-homeomorphism. Let U be $\tau_{1,2}$-open in X. Then U is (1,2)*-αĝ-open in X. Since f^{-1} is (1,2)*-αĝ-irresolute, $(f^{-1})^{-1}$(U) is (1,2)*-αĝ-open in Y. That is f(U) is (1,2)*-αĝ-open in Y. This implies f is (1,2)*-αĝ-open function. Let F be a $\sigma_{1,2}$-closed in Y. Then F is (1,2)*-αĝ-closed in Y. Since f is (1,2)*-αĝ-irresolute, f^{-1}(F) is (1,2)*-αĝ-closed in X. This implies f is (1,2)*-αĝ-continuous function. Hence f is (1,2)*-αĝ-homeomorphism.

4.3. Remark

The following Example shows that the converse of Theorem 4.2 need not be true.

4.4. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*-αĝ-homeomorphism but f is not a strongly (1,2)*-αĝ-homeomorphism.

4.5. Theorem

The composition of two strongly (1,2)*-αĝ-homeomorphisms is a strongly (1,2)*-αĝ-homeomorphism.

Proof

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be two strongly (1,2)*-αĝ-homeomorphisms. Let F be a (1,2)*-αĝ-closed set in Z. Since g is (1,2)*-αĝ-irresolute, g^{-1}(F) is (1,2)*-αĝ-closed in Y. Since f is a (1,2)*-αĝ-irresolute, $f^{-1}(g^{-1}(F))$ is (1,2)*-αĝ-closed in X. That is $(g \circ f)^{-1}$(F) is (1,2)*-αĝ-closed in X. This implies that g o f : (X, τ_1, τ_2) → (Z,

η_1, η_2) is (1,2)*-αĝ-irresolute. Let V be a (1,2)*-αĝ-closed in X. Since f^{-1} is a (1,2)*-αĝ-irresolute, $(f^{-1})^{-1}(V)$ is (1,2)*-αĝ-closed in Y. That is f(V) is (1,2)*-αĝ-closed in Y. Since g^{-1} is a (1,2)*-αĝ-irresolute, $(g^{-1})^{-1}(f(V))$ is (1,2)*-αĝ-closed in Z. That is g(f(V)) is (1,2)*-αĝ-closed in Z. So, (g o f)(V) is (1,2)*-αĝ-closed in Z. This implies that $((g \circ f)^{-1})^{-1}(V)$ is (1,2)*-αĝ-closed in Z. This shows that $(g \circ f)^{-1}$:(Z, η_1, η_2) → (X, τ_1, τ_2) is (1,2)*-αĝ-irresolute. Hence g o f is a strongly (1,2)*-αĝ-homeomorphism.

We denote the family of all strongly (1,2)*-αĝ-homeomorphisms from a bitopological space (X, τ_1, τ_2) onto itself by (1,2)*-sαĝ-h(X).

4.6. Theorem

The set (1,2)*-sαĝ-h(X) is a group under composition of functions.

Proof

By Theorem 4.5, g o f ∈ (1,2)*-sαĝ-h(X) for all f, g ∈ (1,2)*-sαĝ-h(X). We know that the composition of functions is associative. The identity function belonging to (1,2)*-sαĝ-h(X) serves as the identity element. If f ∈ (1,2)*-sαĝ-h(X), then f^{-1} ∈ (1,2)*-sαĝ-h(X) such that f o f^{-1} = f^{-1} o f = I and so inverse exists for each element of (1,2)*-sαĝ-h(X). Hence (1,2)*-sαĝ-h(X) is a group under the composition of functions.

4.7. Theorem

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a strongly (1,2)*-αĝ-homeomorphism. Then f induces an (1,2)*-isomorphism from the group (1,2)*-sαĝ-h(X) onto the group (1,2)*-sαĝ-h(Y).

Proof

Using the function f, we define a function θ_f : (1,2)*-sαĝ-h(X) → (1,2)*-sαĝ-h(Y) by $\theta_f(k)$ = f o k o f^{-1} for every k ∈ (1,2)*-sαĝ-h(X). Then θ_f is a bijection. Further, for all k_1, k_2 ∈ (1,2)*-sαĝ-h(X), $\theta_f(k_1 \circ k_2)$ = f o (k_1 o k_2) o f^{-1} = (f o k_1 o f^{-1}) o (f o k_2 o f^{-1}) = $\theta_f(k_1) \circ \theta_f(k_2)$. Therefore θ_f is an (1,2)*-isomorphism induced by f.

4.8. Remark

The concepts of strongly (1,2)*-αĝ-homeomorphisms and (1,2)*-α-homeomorphisms are independent notions as shown in the following examples.

4.9. Example

Let X = {a, b, c}, τ_1 = {ϕ, X} and τ_2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a, b}} are called $\tau_{1,2}$-open and (1,2)*-α-open; and the sets in {ϕ, X, {c}} are called $\tau_{1,2}$-closed and (1,2)*-α-closed. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, b}} are called (1,2)*-αĝ-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b}}. Then the sets in {ϕ, Y, {a}, {b}, {a, b}} are called $\sigma_{1,2}$-open and (1,2)*-α-open; and the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed and (1,2)*-α-closed in Y. Also the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a strongly (1,2)*-αĝ-homeomorphism but f is not (1,2)*-α-homeomorphism.

4.10. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in X. Moreover, the sets in {ϕ, X, {b}, {c}, {b, c}} are called (1,2)*-α-closed in X and the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called (1,2)*-α-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y} and σ_2 = {ϕ, Y, {a}}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {b}, {c}, {b, c}} are called (1,2)*-α-closed in Y and the sets in {ϕ, Y, {a}, {a, b}, {a, c}} are called (1,2)*-α-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*-α-homeomorphism but not strongly (1,2)*-αĝ-homeomorphism.

4.11. Definition

A bijective function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called (1,2)*-gc-homeomorphism if f is (1,2)*-gc-irresolute and f^{-1} is (1,2)*-gc-irresolute.

4.12. Remark

Theconceptsofstrongly(1,2)*-αĝ-homeomorphisms and (1,2)*-gc-homeomorphisms are independent of each other as the following examples show.

4.13. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in X. Moreover, the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-g-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}} are called (1,2)*-g-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {b}, {a, b}} and σ_2 = {ϕ, Y, {a}, {a, c}}. Then the sets in {ϕ, Y, {a}, {b}, {a, b}, {a, c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-g-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}, {a, c}} are called (1,2)*-αĝ-open and (1,2)*-g-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a strongly (1,2)*-αĝ-homeomorphism but not (1,2)*-gc-homeomorphism.

4.14. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-g-closed in X, and the sets in {ϕ, X, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open and (1,2)*-g-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {a}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-g-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-g-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*-gc-homeomorphism but not strongly (1,2)*-gc-homeomorphism.

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Research Article

Seasonal Variability in Virtual Height of Ionospheric F_2 Layer at the Pakistan Atmospheric Region

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Abstract: The aim of this study was to assess the seasonal variability in virtual height of ionospheric F_2 layer for Pakistan's atmospheric region (PAR). In this communication virtual height variations have been analyzed by the descriptive statistical techniques. These methodologies comprise an autoregressive strategy, linear regression and polynomial regression. The relevance of these models has been illuminated using predicted values of different parameters under the seasonal variation of ionospheric F_2 layer in virtual height that affect the radio wave propagation through the ionosphere. These techniques are implemented to theorize the physical process of varying the virtual heights that leads this study towards formulating the variations due to interaction of radio wave propagation with this ionospheric layer.

Keywords: autoregressive process, variability in virtual height, linear regression, polynomial regression

1. INTRODUCTION

The sun is a main source of ionization in the extraterrestrial atmosphere, so that the variability depends upon the solar activity and geomagnetic conditions. The ionosphere is very important in radio wave communications, which absorbs large quantities of radiant energy from the sun It is earth's upper atmosphere and the F_2 layer is the outer layer of the ionosphere. The virtual height of an ionospheric F_2 layer could be explained as the wave is refracted down gradually rather sharply. Incident and refracted rays below the ionized layer follow the same path as if the reflection has taken place from the greater height. The spot is called virtual height of this layer in other words when the incidence and returned rays are extrapolated to a vertex they meet at a height h' is called virtual height. It has been noted that if the virtual height of the layer is known then it is quite simple to calculate the angle of incidence required for the wave to return to ground at a selected point. The F_2 layer has a permanent existence, even through the height varies on a daily basis. The frequency range of this layer is 3 to 40 MHz. It is the most important reflecting medium for high frequency radio waves and the height of F_2 layer varies from 250- 400 km. Since sun's presence is essential, ionization can take place in any location only during the day time. Even during day time, sun's radiation will not be from morning till evening because sun's zenith angle changes. We may expect maximum ionization at noon, and then falling of both sides of the day's daylight hours. Every event in the sun in respect of thermal content, magnetic field, radiation variation due to sun's rotation around itself should be reflected in the ionospheric properties. The other

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influential factors are latitude, sunspot cycle and magnetic storms [1-3].

The ionosphere seasonal variation is related to a solar zenith angle change, while its solar cycle variation is linked to a change in the solar extreme Ultraviolet (EUV) and x-ray radiation. The important feature is that the maximum electron density (N_mF_2) of F_2 layer is greater in winter than in summer, despite the fact that the solar zenith angle is smaller in summer. Specifically the summer-to-winter neutral circulation results in an increase in the O/N_2 ratio in the winter hemisphere and a decrease in the summer hemisphere. The day-time wavelengths are much shorter then at the night-time. The point is that the day-time electron density of the F_2 layer is very high and it can reflect the higher frequencies. Conversely, the night-time electron density of F_2 layer goes down. Thus, the time delay is used to determine the altitude of reflection and the frequency is an indicator of the electron density at that location. The height which is calculated by assuming that the waves travel with the velocity of light is called the virtual height [4-7].

The different geomagnetic storms can be significantly different and even for a given storm the system's response can be very different in the latitudinal and longitudinal regions. However it is educational to show the ionospheric response to the large magnetic storm that was triggered by a solar flare which appeared at 1229UT on October 19, 1989 and it was 22 maximum cycles. In response to this storm, there were long-lasting electron density depletions at high latitudes as measured DG-Sonde Suparco station Karachi and worldwide Ionosondes. During the incidence of ionosphere disturbances the electron density of the F_2 layer takes a dip and the virtual heights go up due to the heating so that the maximum usable frequencies are decreased. The regular structure of F_2 layer is disrupted and strata appear in it. Through very strong ionosphere disturbances, the ionization of the F_2 layer may drop to a point where the layer will not reflect short waves any longer. The F_2 layer is destroyed during ionosphere disturbances at high geomagnetic Latitudes. We have a recorded data of the virtual height of ionosphere F_2 layer over a period of one year (January to December 1989) which is being used in this study [8-13].

2. MATERIALS AND METHODS

The study was based on the analysis of seasonal variability in virtual height and electron concentration (Ne) for Pakistan's atmospheric region. We have used some statistical tools such as Minitab and Statistica for analyzing time series. For this purpose we have utilized ionospheric data recorded on 256 Digisonde for evolution, analysis and forecasting. Spectral analysis, autocorrelation and partial autocorrelation functions have been effectively implemented on the basis of these statistical software packages.

3. RESULTS AND DISCUSSION

The results presented in this study establish that Auto regressive AR (1) order one character of modeling, the seasonal variability in virtual height of ionospheric F_2 layer as a physical process is an appropriate position. The reported data suggest quantitatively that the seasonal variability in virtual height of ionospheric F_2 layer is occurring Fig. 1 is an original time plot of virtual height of F_2 layer data, Fig. 2 illustrates negative correlation between the electron concentration and the seasonal variability in virtual height of h'F_2 layer Fig. 3. Is a scatter plot of virtual height of h'F_2 layer data. From this graph it appears that the seasonal variability in virtual height of h'F_2 layer in the period (t-1) is useful in predicting the value of the total value of virtual height deliberation in period t. It seems that X_t can be explained as significance of X_{t-1}.To identify the idea we can illustrate the case of autoregressive model which is normally used in time series model.

$$X_t = \alpha_o + \phi X_{t-1} + e_t \quad (1)$$

where, X_t is expressed as a linear combination of its two immediately preceding values, e_t is a error value and α_o is constant (parameters) value

$$X_t = \phi X_{t-1} + \alpha_o \quad (2)$$

Regression analysis describes the assessment of the unknown value of one variable from the known value of the other variable. We have the data which consists of two variables x and y that we want to find the linear function of electron concentration (Ne) of F_2 layer and the virtual height of h'F_2

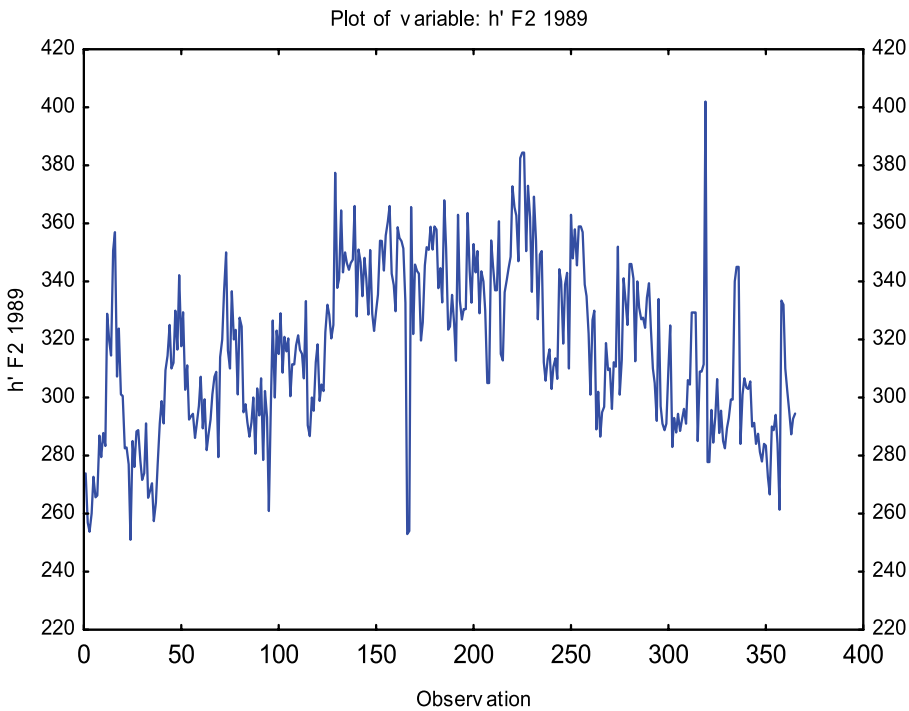
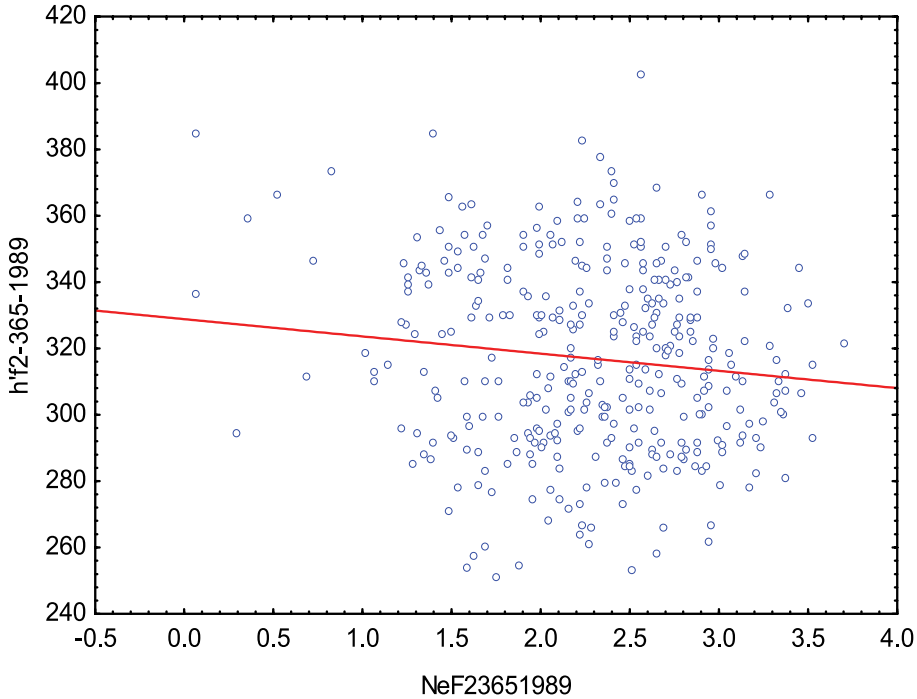


Fig. 1. The temporal variation of virtual height of ionospheric F_2 layer on 19 October 1989.



(Trend equation y = 328.815 -5.1954 +x)

Fig. 2. Correlation between the h'F_2 and Ne.

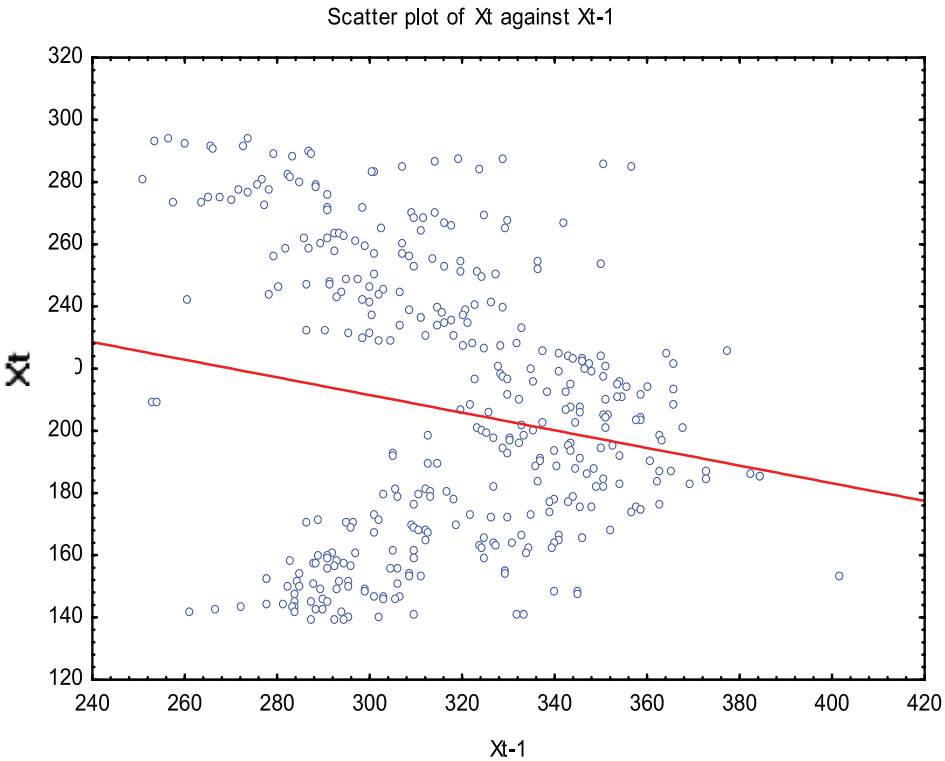


Fig. 3. Virtual height of h'F_2 layer data plotted past values X_{t-1} and present X_t.

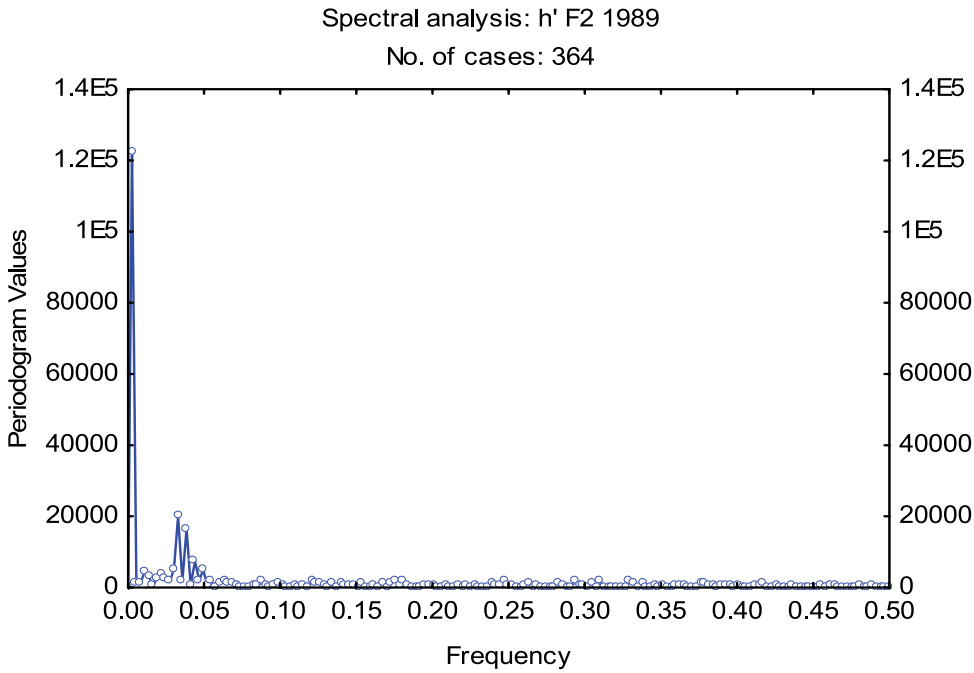


Fig. 4. Periodogram to classify randomness in the seasonal variability of h'F_2.

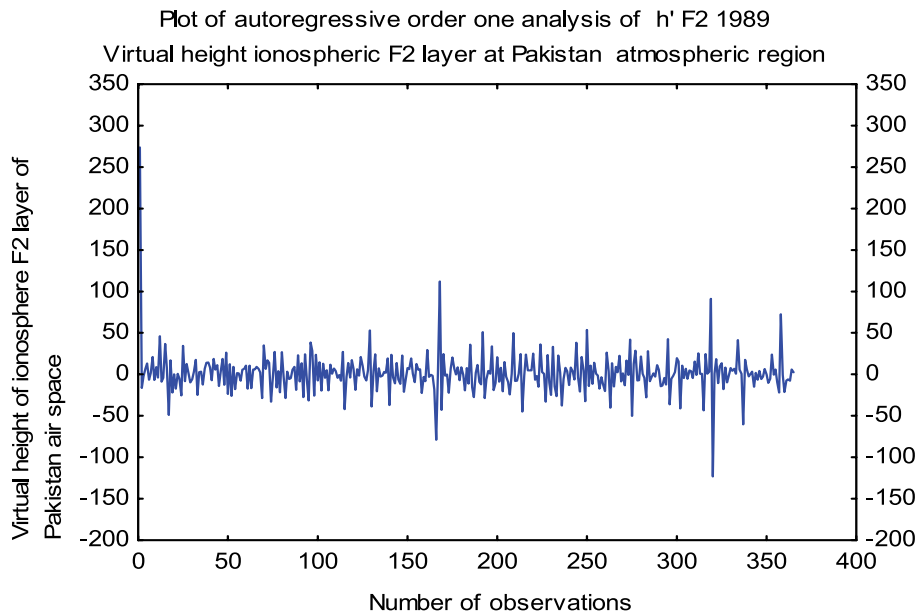


Fig. 5. Residuals analysis of seasonal variability.

layer. The linear regression analysis is the key for building forecasting models to produce forecast of seasonal variability of virtual height of ionosphere h'F_2 layer.

$\overline{X}$ = 296.615 - 0.2837 X_{t-1}

t statistics: (11.421) (12.074)

R^2 = 3.2 %

R^2 is the amount of shared variance between the two variables and useful for the fit of the model.

The periodogram illustrated in Fig. 4 is used to classify randomness in the data series. Also it helps in identifying seasonality in the given time series, and in recognizing the predominance of positive or negative autocorrelation (for positive autocorrelation low-frequency amplitudes should dominate, and for negative autocorrelation, high frequencies should dominate. Thus the following inferences are incurred:

(a) Autoregressive processes may be established – pattern of autocorrelation, of partials, and within the line scale, will show a explanation of a potential model.

(b) The graph of the data set is a visual support to recognize the behavior of the pattern. The autocorrelations and the line scale are the review of the pattern presented in the data. They can expose a great agreement about the data and their characteristics.

(c) The model can be used in the present case to state the dependence between X_t and X_{t-1} in the pair (X_t, X_{t-1}), and to thus relate X_t with X_{t-1}, X_{t-1} with X_{t-} and so on. The plot of X_t and X_{t-1} for t = 2, 3. n = 365 is depicted in Fig. 5. It can be examined that the points are scattered around a straight line. The above model expresses the dependence of the variable on itself at different times for model under consideration α_t at different t are independent, that α_t is independent of α_{t-1}, so that just like ε_t the distribution of α_t is unspecified to be normal. [14, 15]

$$\alpha_t \sim ND(0, \sigma_a^2) \quad (3)$$

Normal (N) distribution (D) with mean zero (0) and variance error term (σ_a^2).It has been distinguished that predictable model is entirely specified only when σ_a^2 is given in calculation to ϕ_1, α_t is understood to be normal. The value of X_t may increase or decrease without bound, because α_t have fixed finite variance and can not continually increase in magnitude to keep X_t within bound as depicted from Fig. 6 that explains the residual analysis specified for this model and also confirms that this model is sufficient.

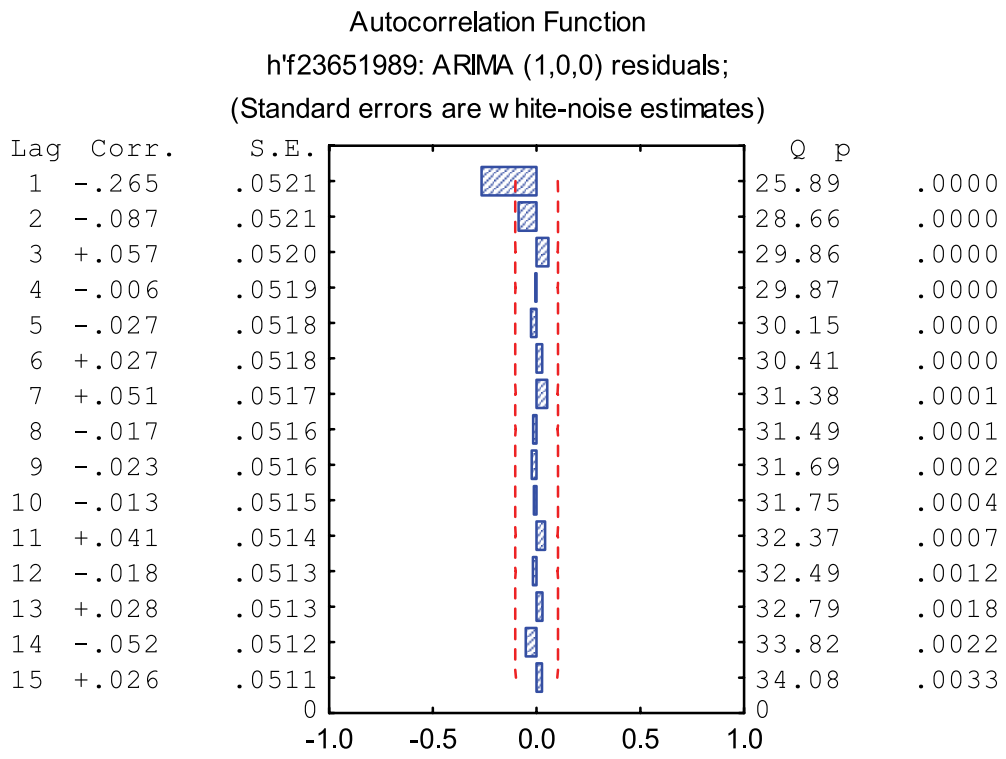


Fig. 6. Plot of Autocorrelation function for seasonal variability.

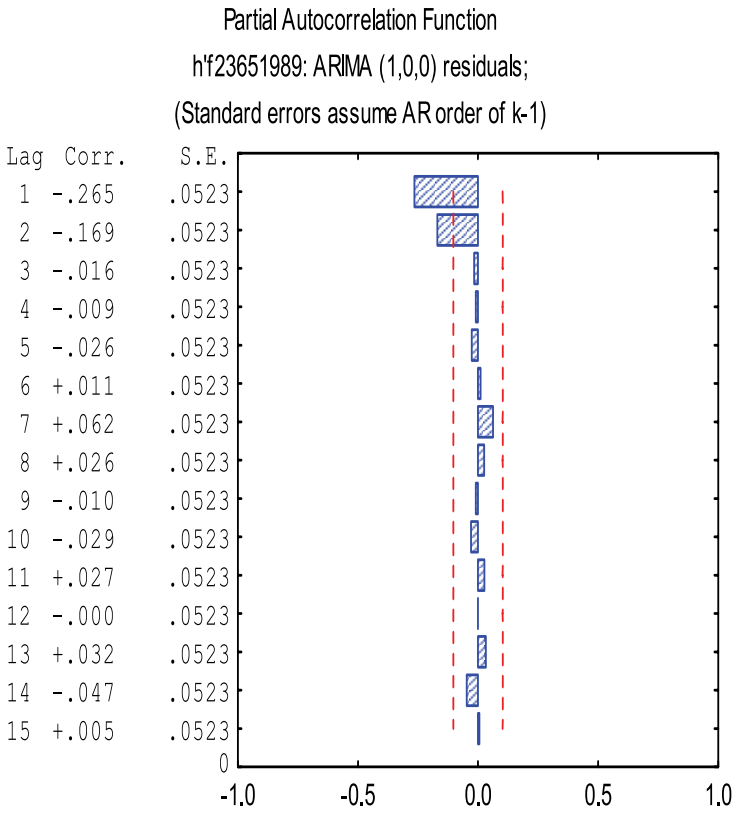


Fig. 7. Partial Autocorrelation function for the residuals of seasonal variability.

Table. 1. Comparison of observed and predicted values.

Days	Observed Values	Predicted values
366	293.89	251.69
367	293.29	233.67
368	292.68	219.74
369	292.68	207.94
370	291.47	197.50
371	290.88	188.03
372	290.28	179.31
373	289.68	171.17
374	289.,85	163.51
375	288.49	156.26

Table. 2. Tests of statistical significance.

S.#.	Model / Test	Simple Regression	Multiple Regression
1	Estimate β_1	-0.126	-0.0283
2	St. Error	25.970	2.340
3	t-Statistics	11.421	- 2.22
4	Constant β_o	325.85	311.213
5	Estimate β_2	-	-0.096
6	MSE	3996.9	1026
7	SSE	3996.9	371443
8	$F_{0..95}$-value	12.074	4.929
9	P – value	0.000573	0.027
10	Corr. Coefficient	Negative	Negative
11	R^2	3.2 %	1.7 %

While a condition tells that if $\phi_1 > 1$ or $\phi_1 < -1$, then the series will be non-stationary or unsteady time series. For a stationary stable time series, X_t remains surrounded in the motive, it has finite variance, we would need $\phi_1 < 1$. Fig. 6 and Fig. 7 illustrate the estimated auto-correlation function and the partial autocorrelation function from lag 1 to 15 respectively. Fig. 6 depicts the estimated correlation between the y–axis vs the lag number on the x-axis and can be used to determine the pattern (AR) in the set of data. Similarly, Fig. 7 shows partial autocorrelation plot for the residuals of the seasonal variability of ionospheric F_2 layer. Partial autocorrelation is used to measure the degree of association between X_t and X_{t-1}, when the effects of other time lags 2, 3, … up to X_{-1} are somehow partial lead out. Their singular purpose in time series analysis is identifying an appropriate AR model for forecasting. For the residuals of the virtual height of ionospheric F_2 layer, that showing an adequacy of the constructed model of seasonal variability in virtual height of F_2 for atmospheric region of Pakistan. The dashed lines in the above mentioned figures demonstrate an approximate 95% confidence interval for an individual estimated partial autocorrelation. In these cases the partial autocorrelation plots strongly recommend the presence of serial correlation in the ionospheric F_2 layer data and measure the degree of association if the process is an autoregressive order one then the partial autocorrelations can be examined to determine the order of the process. In table 1 observed value are plotted against predicted values as depicted in Fig. 8. These illustrations are verifying the results of estimates obtained from the estimating techniques.

Following parametric testes were performed for the model verification through regression and autoregressive AR (1):

1. Standard error test as computed from regression models and autoregressive AR (1) order one model.
2. MSE (Mean Square test). It is a ratio sum of square error and number of sample.
3. SSE test (Sum of the square test). It is called the total sum of the square deviations.
4. t-test (t- distribution test). It is to estimate the confidence interval for the mean of a variable, to make various assumptions about the data.
5. F-test (Fraser test). It is a ratio of the larger variance and the smaller variance.
6. P-value confidence interval (from +95% to -95% confidence level) for models [16, 17].

4. CONCLUSIONS

In this study, we have estimated the seasonal variation of virtual height h' of F_2 layer and the deviation in virtual height of h'F_2 layer due to solar maximum and geomagnetic field. With the predication equation obtained from the time plot, the simple linear regression model may provide a very good fit to data. The autoregressive AR (1) model is clearer and easier to handle than the moving average from the virtual height data set

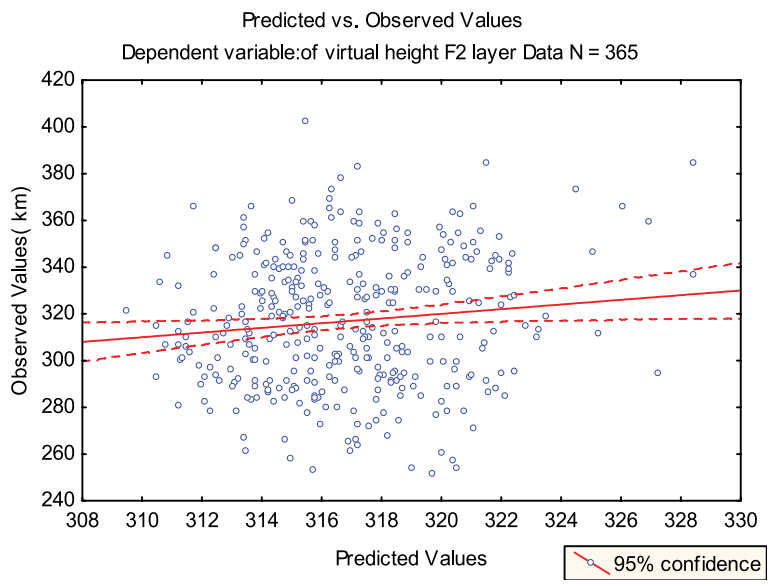


Fig. 8. Comparison between observed and predicted values verifying results of estimates.

which we have analyzed. In the case of finding the appropriate model for seasonal variation in virtual height h'F_2 of ionosphere layer, we have looked into the major parametric values of model. It can be seen that the order one is suitable for making prediction and finding forecasts for the atmospheric region of Pakistan.

5. ACKNOWLEDGEMENTS

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Pakistan Academy of Sciences

Research Article

Extreme Rainfall Incidents over Sindh Province, against Different Return Periods

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Abstract: High frequency and intensity of rainfall events are the major causes of flooding in Pakistan. Statistical distributions, i.e., Gumbell Maximum (GM) distribution & Generalized Extreme Value distribution (GEV), were used to analyze annual extreme rainfalls in different cities of Sindh (Karachi, Badin, Chor, Rohri). Different return periods of extreme annual rainfall were calculated by using these distributions. This article also indicated that GM distribution better explains the yearly incidence of rainfall in Karachi and Badin while the GEV distribution better explains that in Chor and Rohri. We also estimated 200 mm return level for Sindh province in the next 100-year period.

Keywords: Return period, return level, yearly rainfall, Gumbel Maximun Distribution

1. INTRODUCTION

The enhanced greenhouse effect is causing global climate change at a much faster rate compared to its natural pace. There is an increasing change in ambient temperature and rainfall which results an increase in the frequency and intensity of droughts and floods. Pakistan is now experiencing heavy rainfalls and floods due to global warming. This asks for analyzing the pattern of extreme rainfalls all over Pakistan. This study analyzed the extreme rainfalls and its variability over Sindh province of Pakistan, by selecting Karachi (24.54° N, 67.08° E), Badin (24.38° N, 68.58° E), Chor (26.58° N, 69.47° E) and Rohri (27.40° N, 68.90° E) (Fig. 1). Two approaches have been suggested to analyze extreme rainfall [1, 2]. The first step used a quartile method to evaluate the extreme rainfall [2, 3], while in second step statistical distributions have been used to demonstrate yearly extremes with their return periods [4, 5, 6]. Several literatures also depict that log-Pearson type III

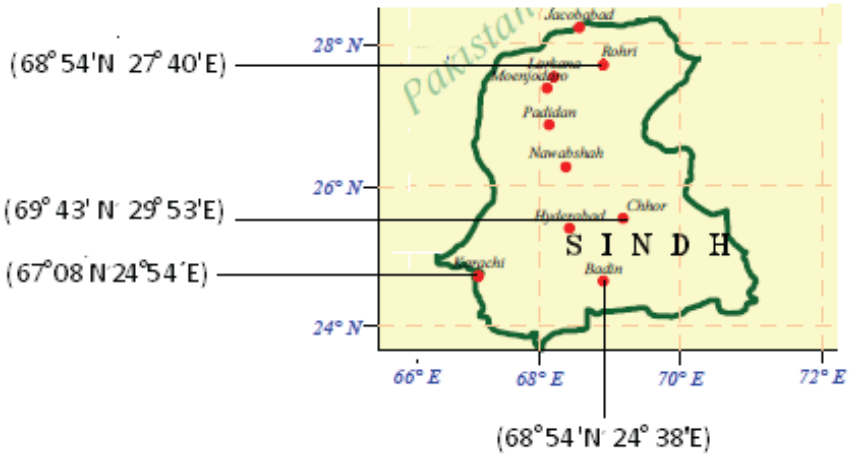

Fig. 1. Geographical location of Rohri, Chor, Karachi & Badin.

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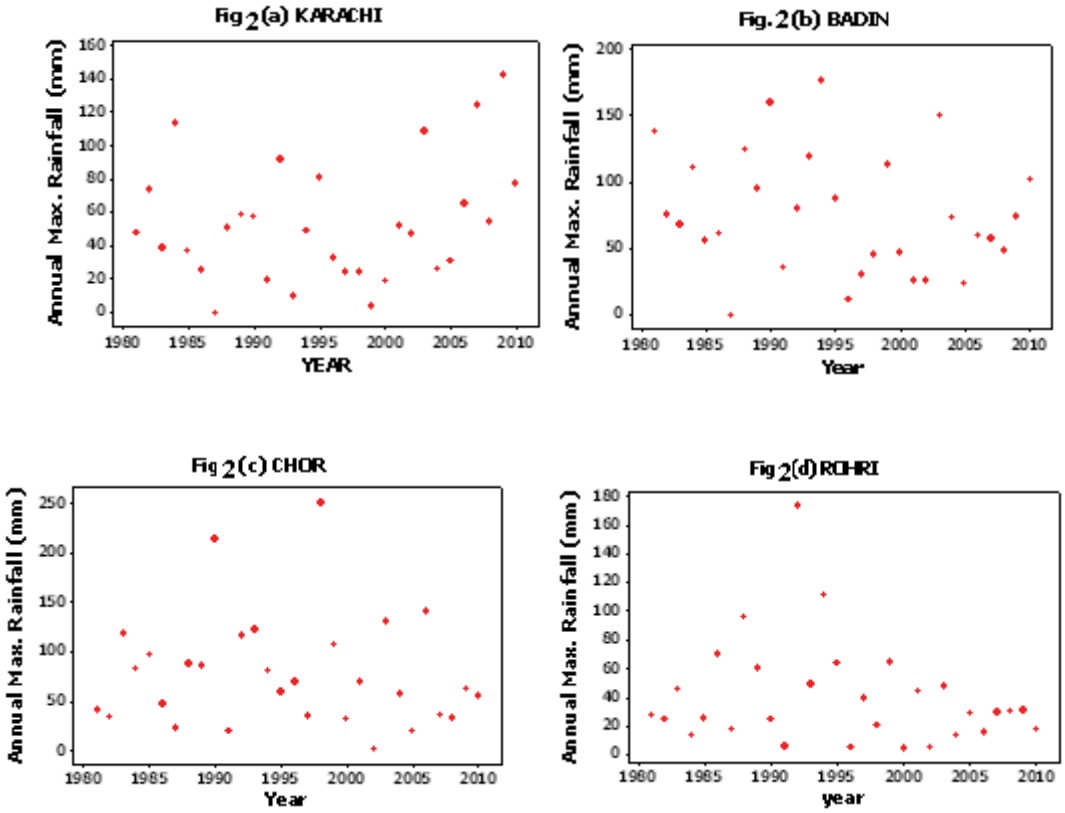

Fig. 2(A). Scatter plots of yearly maximum rainfall.

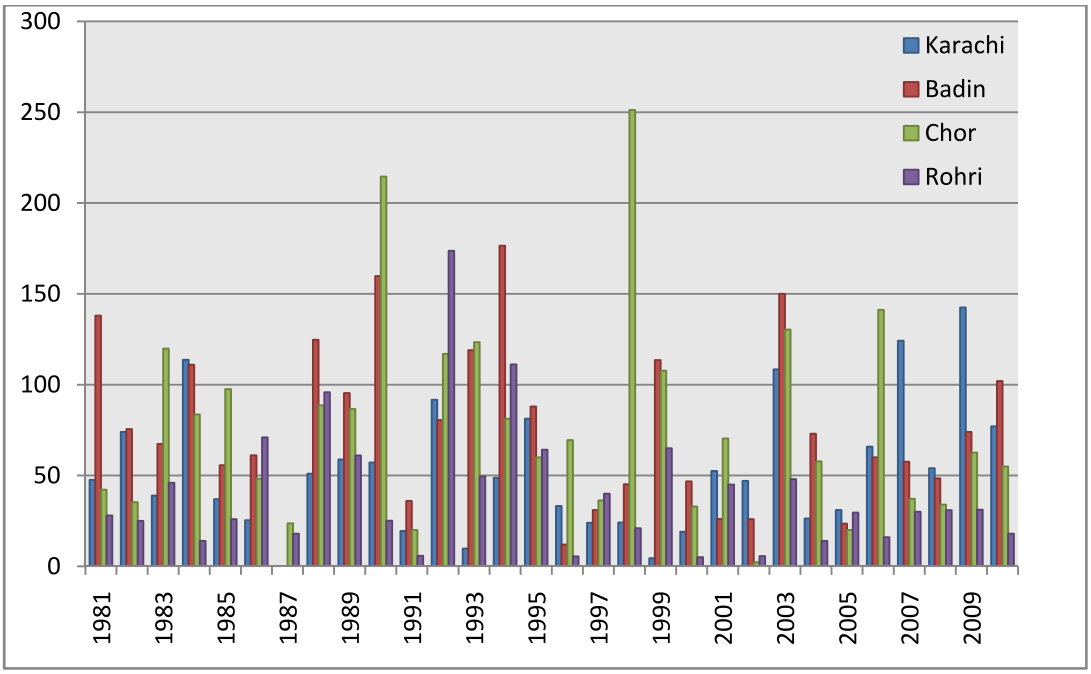

Fig.2(B). Multiple bar diagram of annual extreme rainfall (mm).

analysis, Generalized Extreme Value distribution and Gumbel distribution have been commonly used to examine extreme rainfall events [7]. This study used the second approach to calculate the long return periods and return levels by using the best fit to the extreme value distributions.

2. EXPLORING DAILY RAIN DATA

Daily data from 1981 to the end of 2010, acuired from Pakistan Meteorological Department, has been used for the analysis of annual extreme rainfall over Sindh province. Scatter plots and bar diagram of yearly extreme rainfall values are plotted in Fig. 2(A) and 2(B) respectively. Fig. 2 (A) shows that there is no any observed trend. From Fig.2 (a) we get twelve values more than normal (53 mm) value and yearly maximum rain (142.5 mm) occurred on 19th July 2009, at Karachi station, while it also receives rainfall more than 100 mm i.e. (113.0 mm, 108.4 mm, 124.2 mm and 142.5 mm) at different time intervals, i.e., on 7 Aug, 1984, 29 July 2003,10 Aug 2007 and 19 July 2009, respectively. Fig. 2(b) depicts 12 values greater than normal value (i.e., 76 mm) and Badin receives annual maximum rainfall of 176.5 mm on 3rd Aug 1994, while the other two higher rainfall are above 150 mm per day (i.e., 159.8 mm and 176.5 mm) which occurred on 3 Sep. 1990 and 3 Aug 1994. Fig. 2(c) illustrates the highest frequency of annual rainfall, i.e., it has 13 rainfall incidences of greater than normal value (i.e., 78 mm) and the heaviest rainfall of 251.2 mm was received on 20 Sep 1998; while 2nd, 3rd and 4th highest rainfalls were of 214.6 mm, 141.2 mm and 130.4 mm which occurred on 7 Aug, 1990, 23Aug, 2006 and 26 July, 2003, respectively. Fig. 2 (d) exhibits 11 rainfall incidents of greater than the mean value and the highest rainfall was 173.7 mm on 3 July, 1992, while 2nd, 3rd and 4th highest were 111.2 mm, 95.8 mm and 71 mm which occurred on 3 July 1994, 7 Aug 1988 and 9 Aug 1986. It is obvious that the highest amount of rainfall (251.2 mm per day) occured at Chor, compare to the other stations, during the study period [Fig. 2(B)].

2.1. Extreme Value Distribution

Let $R_1, R_2,\ldots,R_n$ represent daily rainfall within a year. An appropriate model for extremes can be obtained by taking R = max { $R_1, R_2,\ldots,R_n$ }. Where n = 365 and R shows annual maxima. Asymptotic considerations recommend the GEV family [6], who's PDF, is

$$f(R) = \frac{1}{\beta}[1+S]^{1-\frac{1}{k}} \exp\{-[1+S]\}$$

$$1+S > 0 \quad (1)$$

where

$$S = \frac{k(R-\mu)}{\beta} \quad (2)$$

where μ, β and k are Location parameter, scale parameter, and shape parameter, respectively. From analytically integrating Equation (1) we get CDF, i.e.,

$$F(R) = e^{[-(1+S)]^{-\frac{1}{k}}} \quad (3)$$

The inverted CDF gives us the below quintile function.

$$R = F^{-1}(p) = \mu + \frac{\beta}{k}\{[-\ln(P)]^{-k} - 1\} \quad (4)$$

Here F(R) = P shows cumulative probability.

In fitting the GEV distribution, either the method of L-moment is used or maximum likelihood (MLH) method is used [9]. L-moment fitting is favorable for mainly small data samples [8]. The modification of MLH is easy to include additional influences. The special case of GEV distribution which is independent of parameter k, is called Gumbel Distribution which is unbounded. Gumbel found the application of extreme value theory to solve engineering problems and meteorological modeling phenomenon. The PDF of Gumbel distribution is:

$$F(R) = \frac{1}{\beta} e[^{-e^{-S*}} - S*] \quad (5)$$

where

$$S* = \frac{(R-\mu)}{\beta} \quad (6)$$

with location parameter 'μ', scale parameter '**β**' >0 and independent of shape parameters 'k'. Due to its huge utilization to represent the statistics of extremes it is sometimes called "the" extreme value distribution. The Gumbel PDF is rightly skewed and exhibits its maximum at x = μ. CDF of Gumbel distribution is;

$$F(R) = e^{-e^{-S*}} \quad (7)$$

If we invert this CDF, we get quintile function as;

$$R = F^{-1}(p) = \mu - \beta \ln\{-\ln(P)\} \quad (8)$$

The above parameters μ and β can be calculated by using the mean and standard deviation as;

$$\widehat{\beta} = \frac{s\sqrt{6}}{\pi} \tag{9}$$

and

$$\mu = \bar{X} - \gamma\widehat{\beta}, \tag{10}$$

where γ is Euler's constant and whose value is 0.5771.

2.2. Distribution Fitting

This section compares the GEV and Gumbel Maximum distribution for extreme annual rainfall and investigates that which distribution is the best fitted. Here three methods are used.

1) Histogram with fitted probability density function.
2) Chi square test and
3) Probability probability (p-p) plots.

The histogram plots shown in Fig. 3 depict that more area has been covered by GEV distribution as compared to Gumbel Maximum distribution. Thus GEV provides the best fit for yearly extreme rainfalls in Karachi and Badin while Fig. 3(c) and Fig 3(d) show that Gumbel Max provides the best fit for other two cities of Sindh.

Then Chi square test was applied to get the best fitted distribution for extreme annual rainfalls in Karachi, Badin, Chor, and Rohri; the result is summarized in Table 1. As for as Karachi is concerned, the estimated χ^2 = **0.17932** for Gumbel maximum distribution. Under the negative null hypothesis, the statistic is designed from a χ^2 distribution having degree of freedom ν = 6 – 2 – 1 = 3 for Gumbel maximum distribution. representing the ν = 3 of row chi-squared table, expected χ^2 = 0.17932 is smaller than the 95th percentile value i.e. 7.8, so the null hypothesis would be accepted even at 5% level. Thus, the extreme yearly rainfall of Karachi follows Gumbel maximum distribution. Likewise, Table 1 depicts that calculated chi squared values for remaining stations are also smaller than 95th percentile value of 7.8 with 5% significance level. Therefore, the yearly extreme rainfall of four stations follows the Gumbel maximum distribution.

Table 1 Review of goodness of fit test for yearly maximum rainfall in Karachi, Badin, Chor and Rohri.

	Station	Chi-Square Test	
		Generalized Extreme Value Distribution	Gumbel Maximum Distribution
1	Karachi	1.0522	0.17932
2	Badin	0.58497	0.29399
3	Chor	0.20884	0.3613
4	Rohri	0.31646	3.7228

Next, the best fitted test for GEV distribution for which χ^2 is 5.99 at 5% level, with df = ν = 6 – 3 – 1 = 2. Thus, from Table (1) it is clear that the estimated chi squared values for the above stations are smaller than 95th percentile value 5.99 with 5% level of significance. Hence, the yearly extreme values of rainfall of considered stations have GEV distribution. As, calculated values of chi square for Gumbel Maximum are smaller than that of GEV distribution for Karachi and Badin, thus Gumbel maximum is the best fitted distribution for Karachi and Badin, whereas the calculated values of chi square for GEV distribution are smaller that of Gumbel maximum distribution for Chor and Rohri. Therefore GEV distribution is the best fitted for Chor and Rohri.

In order to display the goodness of fit test results p-p Plots are drawn for the yearly maximum rainfall of above cities in Fig. 4. Here we check only the deviation of observed data points from theoretical values. Fig. 4 indicates more deviation in GEV distribution while it is lesser in Gumbel Maximum distribution for Badin and Karachi, and vice versa for Chor and Rohri. So Gumbel Max. is best fitted for Badin and Karachi while GEV is best for Chor and Rohri. Hence, the p-p plots supported the above chi-squared test.

4. RESULTS AND DISCUSSION

This section calculates the annual return period of extreme rainfall of four mentioned cities of Sindh province. The average return period is calculated by the formula:

$$\text{Return } (R) = \frac{1}{\omega[1 - F(R)]} \tag{11}$$

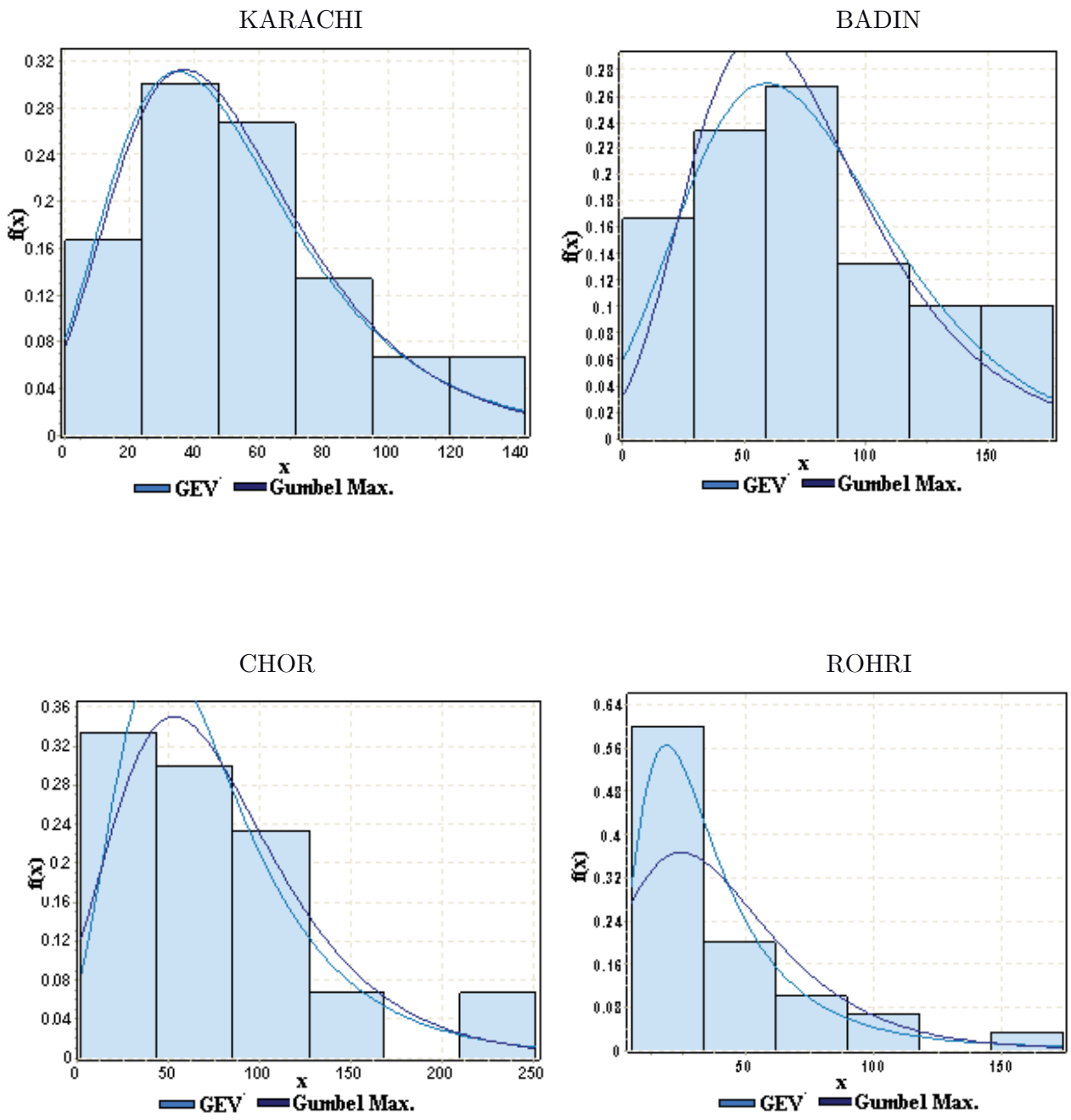

Fig. 3 Comparison of pdf for GEV and Gumbel max. distribution.

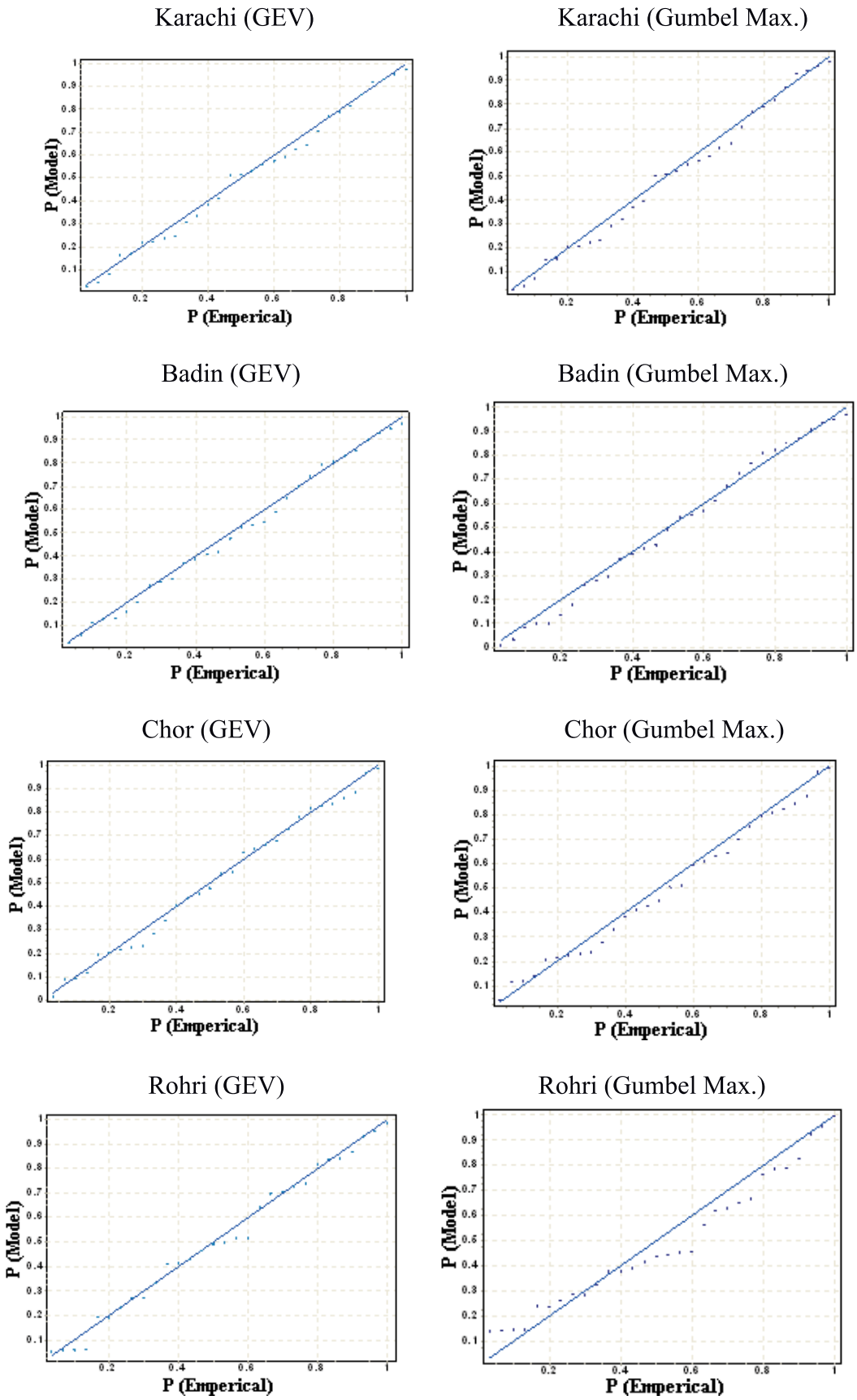

Fig. 4 P-P plot of GEV & Gumbel maximum distribution of four major cities of Sindh.

Table 2 Return levels against given return periods.

Return period (years)	Karachi (mm)	Badin (mm)	Chor (mm)	Rohri (mm)	Average (mm)
2	47.06	68.48	65.27	30.17	52.75
5	78.73	108.55	113.4	57.91	81.13
10	99.7	135.08	149.4	81.55	103.04
20	119.81	160.53	187.39	109.21	130.33
30	134.37	178.96	217.21	132.89	155.05
50	145.85	193.48	242.16	154.04	174.87
100	165.36	218.16	287.77	195.88	200.34

This Eq. represents that the Return (R) is a function of the Cumulative Distribution Function calculated at *R* and the average sampling frequency ω, for yearly maximum value ω = 1/year.

4.1. Return (R) for Karachi

Our calculation reveals that the heaviest rainfall during the period 1981 to 2010, i.e., 142.5 mm, occurred on 19th July, 2009. Obviously, it is desirable to know that when such a high rainfall or even a more heavy rainfall can reoccur in Karachi. So by using Gumbel Distribution (being the best distribution), we calculated the parameter values as μ = 36.814 and β = 27.943.

The cumulative probability is computing using Eq. (6):

$$S^* = \frac{(R - \mu)}{\beta}$$

$S^* = 3.782$,

and using Eq. (7)

$$F(R) = P(R \leq 142.5) = e^{-e^{-S*}}$$

$$F(R) = 0.98$$

Now Eq. (11) gives

$$\text{Return } (R) = \frac{1}{\omega[1 - F(R)]}$$

Return (*R*) = 44 years

This implies that, it will take 44 years to reoccur the rainfall equal to 142.5 mm or more per day. Similarly return period for Chor and Rohri was calculated by using GEV.

4.2. Return Levels against Given Return Periods

This section provides the calculation of return level for $T = 2$ years, with p = 0.5 and F(*X*) = P = 1 − p = 0.5. From Eq. (8), we have 47.06 mm which reveals that in the next two years Karachi may receive 47.06 mm rain approximately. Different return levels 78.73 mm, 99.7 mm, 119.81 mm, 134.37 mm, 145.85 mm and 165.36 mm for Karachi are calculated respectively for return periods, 5, 10, 20, 30, 50 and 100 years.

Hence, return levels are determined against different return period for Karachi and Badin by using Gumbel distribution (Eq. 6, 7 and 8) and for Chor and Rohri by using GEV (Eq. 2, 3 and 4). The calculations are summarized in Table 2. Our results demonstrate that above cities of Sindh have100 mm daily rainfall against the return period of 20-year and 140 mm rainfall against 50-year, suggesting the necessity of appropriate flood forecasting and for acquiring the requisite improvements in river structure in the country.

5. CONCLUSIONS

This study employed probability distributions for the calculation of return periods of yearly rainfall in four major cities of Sindh province of Pakistan, viz. Karachi, Badin, Chor and Rohri. Daily data of rainfall of these cities have been employed from firs Jan 1981 to31 Dec 2010. Analyses of time series of these cities shows no obvious trend.

Our calculations explain that Gumbel maximum distribution provides the best fit for coastal areas of the country, like Karachi and Badin, while the generalized extreme value (GEV) depicts the actual pattern of rainfall in Chor and

Rohri. Our results also revealed that the considered cities of Sindh will have 130 mm return levels against 20-year return period, while rainfall of 170 mm or more per day is expected in the next 50-year return period. So it suggests for upgrading the flood forecasting system, by using modern technology like GIS, improving the river flows, and constructing new dams in Pakistan.

6. ACKNOWLEDGEMENTS

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Research Article

Properties of Univalent Solution for Complex Fractional Differential Equation

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Abstract: In this note, we discuss some properties of univalent solutions for fractional differential equation in the unit disk in the sense of Srivastava-Owa operators. By employing the differential subordination concept, we will study the upper bound of these solutions. Furthermore, by applying the Rogosinski theorem and Goluzin theorem, we illustrate some inequalities involving the coefficients and integral representation of these solutions. Moreover, the uniqueness is studied by using Rouche's theorem.

Keywords: Fractional calculus; fractional differential equation; unit disk; univalent function; analytic function; subordination; superordination

AMS Mathematics Subject Classification: 30C45.

1. INTRODUCTION

In [1], Srivastava and Owa, gave definitions for fractional operators (derivative and integral) in the complex z-plane $\mathbb{C}$ as follows:

Definition 1.1 The fractional derivative of order α is defined, for a function $f(z)$ by

$$D_z^{\alpha} f(z) := \frac{1}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^{\alpha}} d\zeta;\ 0 \le \alpha < 1,$$

where the function $f(z)$ is analytic in simply-connected region of the complex z-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z-\zeta)^{-\alpha}$ is removed by requiring $log(z-\zeta)$ to be real when $(z-\zeta) > 0$.

Definition 1.2 The fractional integral of order α is defined, for a function $f(z)$, by

$$I_z^{\alpha} f(z) := \frac{1}{\Gamma(\alpha)} \int_0^z f(\zeta)(z-\zeta)^{\alpha-1} d\zeta;\ \alpha > 0,$$

where the function $f(z)$ is analytic in simply-connected region of the complex z-plane $(\mathbb{C})$ containing the origin and the multiplicity of $(z-\zeta)^{\alpha-1}$ is removed by requiring $log(z-\zeta)$ to be real when $(z-\zeta) > 0$.

Let $\mathbb{A}$ denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in U. \tag{1}$$

Also, let $\mathbb{S}$ and $\mathbb{C}$ denote the subclasses of $\mathbb{A}$ consisting of functions which are, respectively, univalent and convex in U. It is well known that; if the function $f(z)$ given by (3) is in the class $\mathbb{S}$, then $|a_n| \le n, \quad n \in \mathbb{N} \setminus \{1\}$. Moreover, if the

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function $f(z)$ given by (1) is in the class $\mathbb{C}$, then $|a_n| \le 1, \quad n \in \mathbb{N}$.

Definition 1.3 (Subordination Principal). For two functions f and g analytic in U, we say that the function $f(z)$ is subordinate to $g(z)$ in U and we write as $f(z) \prec g(z)(z \in U)$, if there exists a Schwarz function $w(z)$ analytic in U with $w(0) = 0$, and $|w(z)| < 1$, such that $f(z) = g(w(z)), z \in U$. In particular, if the function $g(z)$ is univalent in U, the above subordination is equivalent to $f(0) = g(0)$ and $f(U) \subset g(U)$.

Definition 1.4 (Differential subordination) Let $\phi : \mathbb{C}^2 \to \mathbb{C}$ and let h be univalent in U. If p is analytic in U and satisfies the differential subordination $\phi(p(z)), zp'(z)) \prec h(z)$ then p is called a solution of the differential subordination. The univalent function q is called a dominant of the solutions of the differential subordination, if $p \prec q$. If p and $\phi(p(z)), zp'(z))$ are univalent in U and satisfy the differential superordination $h(z) \prec \phi(p(z)), zp'(z))$, then p is called a solution of the differential superordination. An analytic function q is called subordinant of the solution of the differential superordination if $q \prec p$.

In [2] the authors imposed a linear fractional differential operator based on $D_z^{\alpha} f(z)$ as follows:

$$\begin{aligned}
D^0 f(z) &= f(z) \\
&= z + \sum_{n=2}^{\infty} a_n z^n, \\
D_{\beta,\lambda}^{1,\alpha} f(z) &= (\beta - \lambda)\Phi^{\alpha} f(z) + \lambda z(\Phi^{\alpha} f(z))' + (1-\beta)z \\
&= z + \sum_{n=2}^{\infty} [\frac{\Gamma(n+1)\Gamma(2-\alpha)}{\Gamma(n+1-\alpha)}][\lambda(n-1)+\beta] a_n z^n \\
&= D_{\beta,\lambda}^{\alpha}(f(z)), \\
D_{\beta,\lambda}^{2,\alpha} f(z) &= D_{\beta,\lambda}^{\alpha}(D_{\beta,\lambda}^{1,\alpha} f(z)) \\
&= z + \sum_{n=2}^{\infty} \{[\frac{\Gamma(n+1)\Gamma(2-\alpha)}{\Gamma(n+1-\alpha)}][\lambda(n-1)+\beta]\}^2 a_n z^n, \\
&\vdots \\
D_{\beta,\lambda}^{k,\alpha} f(z) &= D_{\beta,\lambda}^{\alpha}(D_{\beta,\lambda}^{k-1,\alpha} f(z)) \\
&= z + \sum_{n=2}^{\infty} \{[\frac{\Gamma(n+1)\Gamma(2-\alpha)}{\Gamma(n+1-\alpha)}][\lambda(n-1)+\beta]\}^k a_n z^n \\
&:= z + \sum_{n=2}^{\infty} \Psi_{n,k}(\alpha,\beta,\lambda) a_n z^n,
\end{aligned} \tag{2}$$

for $0 \le \alpha < 1, \beta \ge 1, \lambda \ge 0$ and $k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ with $D_{\beta,\lambda}^{k,\alpha} f(0) = 0$.

Corresponding to the differential operator $D_{\beta,\lambda}^{k,\alpha}$, the fractional integral operator is given as follows

$$I_{\beta,\lambda}^{k,\alpha} f(z) = z + \sum_{n=2}^{\infty} \frac{a_n z^n}{\{[\frac{\Gamma(n+1)\Gamma(2-\alpha)}{\Gamma(n+1-\alpha)}][\lambda(n-1)+\beta]\}^k}$$

and hence we obtain

$$I_{\beta,\lambda}^{k,\alpha} D_{\beta,\lambda}^{k,\alpha} f(z) = f(z).$$

Remark 1.1. The operator (2) generalizes various types of operators such as S ă l ă gean ' s differential operator [3], Al-Oboudi ' s differential operator [4], Srivastava-owa fractional differential operator, the linear multiplier fractional differential operator which introduced in [5]. Finally, when $\beta = 1$, we obtain the linear fractional operator as in [6].

Here, we consider the following class of fractional differential equation:

$$\frac{z(D_{\beta,\lambda}^{k,\alpha} f(z))'}{D_{\beta,\lambda}^{k,\alpha} f(z)} = p(z) \tag{3}$$

such that $p(z) \prec q(z)$ and $q(0) = p(0) = 1$. We denote this class by $S_{\beta,\lambda}^{k,\alpha}(q)$.

We need the following preliminaries in the sequel.

The Libera-Pascu integral operator $L_a : \mathbb{A} \to \mathbb{A}$ defined by

$$F(z) := L_a f(z) = \frac{1+a}{z^a} \int_0^z f(t) t^{a-1} dt, a \in \mathbb{C}, \Re(a) \ge 0.$$

For $a = 1$ we obtain the Libera integral operator, for $a = 0$ we obtain the Alexander integral operator and in the case $a = 1,2,3,...$ we obtain the Bernardi integral operator.

Lemma 1.1 [7] Let h be convex in U and θ, ϕ be analytic in domain D. Let p be analytic in U, with $h(0) = \theta(p(0))$ and $p(U) \subset D$. If the differential equation

$$\theta[q(z)] + zq'(z)\phi[q(z)] = h(z)$$

has a univalent solution in U that satisfies $q(0)=p(0)$ and $\theta[q(z)] \prec h(z)$ then the differential subordination

$$\theta[p(z)]+zp'(z)\phi[p(z)] \prec h(z)$$

implies that $p(z) \prec q(z)$. The function q is the best dominant.

Lemma 1.2 (Rogosinski Theorem) [8] Let $f(z)=\sum_{n=1}^{\infty} a_n z^n$ and $g(z)=\sum_{n=1}^{\infty} b_n z^n$ be analytic in U and suppose $g \prec f$. Then

$$\sum_{n=1}^{k} |b_n|^2 \le \sum_{n=1}^{k} |a_n|^2 .$$

Lemma 1.3 (Goluzin Theorem) [8] If $g \prec f$ and $\lambda_1 \ge \lambda_2 \ge \ldots \ge 0$, then

$$\sum_{n=1}^{\infty} \lambda_n |b_n|^2 \le \sum_{n=1}^{\infty} \lambda_n |a_n|^2 .$$

2. MAIN RESULTS

Our main results are provided in this section.

Theorem 2.1 Let $f \in S_{\beta,\lambda}^{k,\alpha}(q)$,

$$P(z) := \frac{z(D_{\beta,\lambda}^{k,\alpha} F(z))'}{D_{\beta,\lambda}^{k,\alpha} F(z)},$$

where F is the Libera-Pascu integral operator, be analytic in U such that $P(U) \subset D$ (a domain) and

$$h(z)=\left(\frac{a}{1+a}q(z)+\frac{q^2(z)}{1+a}+\frac{zq'(z)}{1+a}\right)\varphi(z), \quad \varphi(0)=1$$

has a univalent solution in the unit disk U such that $P(0)=q(0)$. Then the subordination

$$\left(\frac{a}{1+a}P(z)+\frac{P^2(z)}{1+a}+\frac{zP'(z)}{1+a}\right)\phi(z) \prec$$
$$\left(\frac{a}{1+a}q(z)+\frac{q^2(z)}{1+a}+\frac{zq'(z)}{1+a}\right)\phi(z)$$

implies $P(z) \prec q(z)$ for some analytic function φ and q is the best dominant.

Proof. Since $f \in S_{\beta,\lambda}^{k,\alpha}(q)$, then

$$\frac{z(D_{\beta,\lambda}^{k,\alpha} f(z))'}{D_{\beta,\lambda}^{k,\alpha} f(z)} \prec q(z).$$

From the definition of the Libera-Pascu integral operator we have

$$(1+a)f(z)=aF(z)+zF'(z),$$

and by using the linear operator $D_{\beta,\lambda}^{k,\alpha}$, we have

$$(1+a)D_{\beta,\lambda}^{k,\alpha} f(z)=aD_{\beta,\lambda}^{k,\alpha} F(z)+$$
$$D_{\beta,\lambda}^{k,\alpha}(zF'(z)), \quad \Re(a) \ge 0.$$

By making use the first derivative of the last assertion, we obtain

$$z(D_{\beta,\lambda}^{k,\alpha} f(z))'=\frac{a}{(1+a)}z(D_{\beta,\lambda}^{k,\alpha} F(z))'$$
$$+\frac{1}{(1+a)}z[D_{\beta,\lambda}^{k,\alpha}(zF'(z))]'$$

and using the fact that

$$D_{\beta,\lambda}^{k,\alpha}(zF'(z))=z(D_{\beta,\lambda}^{k,\alpha} F(z))',$$

yields

$$\begin{aligned} z(D_{\beta,\lambda}^{k,\alpha} f(z))' &= \frac{a}{(1+a)}z(D_{\beta,\lambda}^{k,\alpha} F(z))'+\frac{1}{(1+a)}z[z(D_{\beta,\lambda}^{k,\alpha}(F(z))']' \\ &= \left\{\frac{\frac{a}{(1+a)}z(D_{\beta,\lambda}^{k,\alpha} F(z))'}{D_{\beta,\lambda}^{k,\alpha} F(z)}+\frac{\frac{1}{(1+a)}z[z(D_{\beta,\lambda}^{k,\alpha}(F(z))']'}{D_{\beta,\lambda}^{k,\alpha} F(z)}\right\}D_{\beta,\lambda}^{k,\alpha} F(z) \\ &= \left(\frac{a}{1+a}P(z)+\frac{P^2(z)}{1+a}+\frac{zP'(z)}{1+a}\right)D_{\beta,\lambda}^{k,\alpha} F(z) \end{aligned}$$

A computation implies that

$$\frac{z(D_{\beta,\lambda}^{k,\alpha} f(z))'}{D_{\beta,\lambda}^{k,\alpha} f(z)} = \theta[P(z)]+zP'(z)\phi(P(z))$$

where

$$\theta[P(z)] := \left(\frac{a}{1+a}P(z)+\frac{P^2(z)}{1+a}\right)\varphi(z),$$

$$\phi(P(z)) := \frac{\varphi(z)}{1+a}$$

and

$$\varphi(z) := \frac{D_{\beta,\lambda}^{k,\alpha} F(z)}{D_{\beta,\lambda}^{k,\alpha} f(z)}$$

are analytic in U. It is clear that $\theta[q(z)] \prec h(z)$. Hence in view of Lemma 1.1, we have $P(z) \prec q(z)$ and q is the best dominant.

Corollary 2.1 Let the assumptions of Theorem 2.1 hold. Then the Libera-Pascu integral operator $F(z) \in S_{\beta,\lambda}^{k,\alpha}(q)$.

Next, using Lemma 1.2 and Lemma 1.3, we introduce the coefficient inequalities as follows.

Theorem 2.2 Let $f \in S_{\beta,\lambda}^{k,\alpha}(q)$. Then

$$\sum_{n=1}^{k} |b_n|^2 + 1 \le \sum_{n=1}^{k} |q_n|^2 + 1, \tag{4}$$

where $q(z) = \sum_{n=0}^{\infty} q_n z^n, q(0) = 1$ and $b_1 := \Psi_{2,k}(\alpha,\beta,\lambda) a_2, b_2 := 2\Psi_{3,k}(\alpha,\beta,\lambda) a_3 - [\Psi_{2,k}(\alpha,\beta,\lambda) a_2]^2, \ldots$

Proof. Since $f \in S_{\beta,\lambda}^{k,\alpha}(q)$, then we have

$$\frac{z(D_{\beta,\lambda}^{k,\alpha} f(z))'}{D_{\beta,\lambda}^{k,\alpha} f(z)} \prec q(z).$$

A computation implies that

$$\begin{aligned}\frac{z(D_{\beta,\lambda}^{k,\alpha} f(z))'}{D_{\beta,\lambda}^{k,\alpha} f(z)} &= 1 + \Psi_{2,k}(\alpha,\beta,\lambda) a_2 z \\ &+ \{2\Psi_{3,k}(\alpha,\beta,\lambda) a_3 \\ &- [\Psi_{2,k}(\alpha,\beta,\lambda) a_2]^2\} z^2 + \ldots \\ &\prec 1 + q_1 z + q_2 z^2 + \ldots.\end{aligned}$$

which is equivalent to

$$\begin{aligned}&1 + \Psi_{2,k}(\alpha,\beta,\lambda) a_2 z + \{2\Psi_{3,k}(\alpha,\beta,\lambda) a_3 \\ &- [\Psi_{2,k}(\alpha,\beta,\lambda) a_2]^2\} z^2 + \ldots \prec 1 + q_1 z + q_2 z^2 + \ldots.\end{aligned}$$

or

$$\begin{aligned}&\Psi_{2,k}(\alpha,\beta,\lambda) a_2 z + \{2\Psi_{3,k}(\alpha,\beta,\lambda) a_3 \\ &- [\Psi_{2,k}(\alpha,\beta,\lambda) a_2]^2\} z^2 + \ldots \prec q_1 z + q_2 z^2 + \ldots.\end{aligned}$$

thus in view of Lemma 1.2

$$\sum_{n=1}^{k} |b_n|^2 \le \sum_{n=1}^{k} |q_n|^2,$$

and consequently we obtain (4).

As applications of Lemma 1.3, we have the following theorems:

Theorem 2.3 Let $f \in S_{\beta,\lambda}^{k,\alpha}(q)$. Then for $m\lambda + \beta > 0, \quad m = 1,2,3,..,$

$$\sum_{n=1}^{\infty} \lambda_n |b_n|^2 + 1 \le \sum_{n=1}^{\infty} |q_n|^2 \lambda_n + 1, \tag{5}$$

where

$$\begin{aligned}\lambda_1 &= \left(\frac{2(\lambda+\beta)}{2-\alpha}\right)^k \\ \lambda_2 &= \left(\frac{6(2\lambda+\beta)}{(2-\alpha)(3-\alpha)}\right)^k \\ \lambda_3 &= \left(\frac{24(3\lambda+\beta)}{(2-\alpha)(3-\alpha)(2-\alpha)}\right)^k \\ &\vdots,\end{aligned}$$

such that $\lambda_1 \ge \lambda_2 \ge \lambda_3 \ge \ldots \ge 0.$

Theorem 2.4 Let $f \in \mathsf{A}$ such that $|a_1| \le |a_2| \le \ldots$. Assume that the solution of the fractional differential equation

$$D_{\beta,\lambda}^{k,\alpha} f(z) = g_\alpha(z) \tag{6}$$

satisfying the subordination $f(z) \prec G(z),\ G(0) = 0.$ Then the solution is bounded.

Proof. The solution of the equation (6) has the form

$$f(z) = I_{\beta,\lambda}^{k,\alpha} g_\alpha(z).$$

Since $f(z) \prec G(z)$, then Lemma 1.3 implies

$$\begin{aligned}|f(z)| = \left|\sum_{n=1}^{\infty} a_n z^n\right| &\le \sum_{n=1}^{\infty} |a_n|, \quad (a_1 = 1) \\ &= \sum_{n=1}^{\infty} \frac{|a_n|^2}{|a_n|} \le \sum_{n=1}^{\infty} \frac{|g_n|^2}{|a_n|}.\end{aligned}$$

But $\frac{1}{|a_1|} \ge \frac{1}{|a_2|} \ge \ldots$ hence the solution $f(z)$ is bounded.

Theorem 2.5 If

$$\sum_{n=1,n\neq k}^{\infty} |a_n| \leq |a_k|, \quad a_k \neq 0, k \geq 2.$$

Then Eq. (3) has a unique solution in the unit disk.

Proof. By setting

$$\phi(z) = \frac{-1}{a_k} \sum_{n=1,n\neq k}^{\infty} a_n z^n, \quad k \geq 2$$

for $|z| \leq 1$, implies

$$\begin{aligned} |\phi(z)| &= \left|\frac{-1}{a_k} \sum_{n=1,n\neq k}^{\infty} a_n z^n\right| \\ &= \left|\frac{1}{a_k}\right| \left|\sum_{n=1,n\neq k}^{\infty} a_n z^n\right| \\ &\leq \left|\frac{1}{a_k}\right| \sum_{n=1,n\neq k}^{\infty} |a_n| \\ &< 1. \end{aligned}$$

Since $|\phi(z)| < 1$ for $|z| = 1$, and by Rouche's theorem, we observe that (3) has exactly one zero in U.

Theorem 2.6 Let the assumption of Theorem 2.4 hold. If for positive integer m

$$\sum_{n=1}^{\infty} |g_n|^2 < |a_m| \tag{7}$$

then the fractional differential equation (6) has a unique solution in U.

Proof. From the Proof of Theorem 2.4, we have

$$|f(z)| \leq \sum_{n=1}^{\infty} \frac{|g_n|^2}{|a_n|}.$$

By the assumption of the theorem, there exists a coefficient a_m such that $1/|a_n| \leq 1/|a_m|$ for all n. Therefore by (7), we obtain

$$\begin{aligned} |f(z)| &\leq \left|\frac{1}{a_m}\right| \sum_{n=1}^{\infty} |g_n|^2 \\ &< 1; \end{aligned}$$

thus by Rouche's theorem, Eq.(6) has a unique solution.

Theorem 2.7 Let the assumption of Theorem 2.6 hold. If

$$\sum_{n=1}^{\infty} |g_n|^2 < 1, \tag{8}$$

then the fractional differential equation (6) has a unique solution in U.

Proof. Since

$$|f(z)| \leq \sum_{n=1}^{\infty} \frac{|g_n|^2}{|a_n|}.$$

Then we impose

$$\begin{aligned} |f(z)| &\leq \sum_{n=1}^{\infty} \frac{|g_n|^2}{|a_n|} \\ &\leq \sum_{n=1}^{\infty} \frac{1}{|a_1|} |g_n|^2 \\ &= \sum_{n=1}^{\infty} |g_n|^2 \\ &< 1; \end{aligned}$$

hence by Rouche's theorem, Eq.(6) has a unique solution.

3. CONCLUSIONS

By employing a linear fractional differential operator in the unit disk in the sense of the Srivastava-Owa differential operator, fractional differential equations (3) and (6) are introduced. In Theorem 2.1, we considered that the Eq. (3) has a univalent solution. The important properties of this solution are described by making use of the Libera-Pascu integral operator, subordination concept, differential subordination and coefficients bound. Theorems 2.5-2.7 are proposing the unique solution of Eq.(3) and (6) in the unit disk by applying the Rouche's theorem. Theorem 2.6 showed the connection between the existence of unique solution and the coefficients bound. Similarly for Theorem 2.7.

4. ACKNOWLEDGEMENT

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Pakistan Academy of Sciences

Obituary

Professor Dr. Mustafa Shameel

(1941 – 2013)

We are grieved to report the sad demise of our friend and colleague, Prof. Dr. Mustafa Shameel, an eminent Meritorious Professor of Botany, University of Karachi, on 13th May 2013 at his residence, Ouj-e-Suraiya, A-338, Block 7, Gulistan-e-Jauhar, Karachi-75290, Pakistan. He had been suffering from multiple ailments for the last few years, but remained active and delightfully engaged in shaping papers of his students until he last breath. This reflects his keen infatuation for phycological research.

Prof. Dr. Mustafa Shameel was an authority on botany, biology and phycochemistry of seaweeds. He was the developer of a new classification of algae, and founder and patron of *International Journal of Phycology & Phycochemistry.*

Prof. Dr. *Mustafa Shameel* was born on 3rd July, 1941 at Rudauli (UP, British India) as the younger son of Mr. *Syed Amirul Hasan Quadri*. He obtained his MSc in Botany from the University of Karachi in 1962 by securing 1st division and 2nd position and PhD in Marine Botany in 1972 from the University of Kiel, Germany under German Academic Exchange Scholarship. In 1977, he completed 14-month Post-Doctoral research from the same University under Alexander von Humboldt Fellowship. Also, he did MDH in 1963 with 1st division, MBSc with 1st division in 1964 and MSc (H) with Gold Medal in Homoeopathy from International Medical College Lahore, affiliated with Nature Cure Institute, Bradford England.

Dr Shameel served at the Department of Botany, University of Karachi as Lecturer (1962-1972), Assistant Professor (1972-1973), Associate Professor (1978-1979), Professor (1985-1999), Meritorious Professor (1999-2001) and HEC Eminent Professor (2003-2005). He also worked as Director, Institute of Marine Science (1994-1998) and Centre of Excellence in Marine Biology (1999-2001), University of Karachi. He was the President of Pakistan Botanical Society during 2001-2003.

Prof. Dr. Shameel was elected as Fellow of Pakistan Academy of Sciences in 2005 and as Fellow of The Academy of Sciences for the Developing World, Trieste, Italy (TWAS) in 2008.

Prof Shameel had more than 50-year experience of teaching and research. He was a globally recognized authority on marine benthic algae. He published more than 330 scientific research papers in journals of international repute and 7 books with emphasis on marine algae of the Baltic, North, Mediterranean & Arabian seas. He participated in several national and international conferences and presented scientific papers. He supervised research work of 17 PhD students, 15 from Botany and two from Marine Biology. Dr. Shameel presented a new concept of barobiology of seaweeds in 1972 and reported for the first time the effects of high hydrostatic pressure on the physiology and biochemistry of marine algae. Apart from taxonomic accounts of 28 new taxa of marine algae, he has emphasized on seaweed natural products, where 24 new compounds such as sterols, terpenes, glycosides and other secondary metabolites have been isolated and described. Six new species of algae have been named after him like *Ulothrix shameelii* Faridi *in* Faridi MAF, Anjum G & Haq 1 1982:183 Pak J Bot 14 (2): 181-188, *Scinaia shameelii* Afaq-Husain S 1996: 446 Candollea 51 (2): 445-459, *Stypopodium shameelii* Nizamuddin M & Aisha K 1996:128 Pak J Bot 28 (2): 127-141, *Codium shameelii* Nizamuddin M 1997:179 Pak J Bot 29 (2): 179-184

and *Ceratium shameelii* Ghazala B 2009:55 Int J Phycol Phycochem 5(1): 55-56. He developed a new concept of phycochemistry in 1990, proposed a new classification of algae by creating 22 new taxonomic groups in 2001, modified in 2008 and revised it in 2012. After his retirement in 2001, he spent an amount of Rs. 2,25,000/- from his provident fund to lay down the foundation of *International Journal of Phycology and Phycochemistry* (IJPP) in 2005 as a biannual publication.

Dr. Shameel has been a member of the Editorial Board of Pakistan Journal of Botany, Pakistan Journal of Marine Sciences, Marine Research, Biological Research Journal and Journal of Natural History & Wildlife and FUUAST Journal of Biology. He served as Editor-in-Chief of Pakistan Journal of Marine Biology (1999-2001), and as Chief Editor of International Journal of Phycology & Phycochemistry (2005-2010). He received "Meritorious Research Award" of the University of Karachi (1997) and "Agha Hasan Abedi Gold Medal" in Botany from Pakistan Academy of Sciences (1998). He was assigned 10th National Ranking in Biology in 1999 as well as 2000, by Pakistan Council for Science & Technology. In recognition of his outstanding contributions to scientific research, Prof. Shameel was awarded "*Izaz-i-Fazeelat*" on 14th August 2000 and "*Tamga-i-Imtiaz*" on 14th August 2001 by the President of Pakistan.

Professor Shameel had a charming and amiable personality and had many friends among the scientific community. May Allah the Almighty bestow his countless blessings upon him and strength to his numerous students, colleagues, friends and members of his family within the country and abroad to bear this irreparable loss. Ameen!

Prof. Dr. Viqar Uddin Ahmad, *HI, SI*

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CONTENTS

Volume 50, No. 2, June 2013 **Page**

Research Articles

Engineering Sciences

Parabolic Trough Solar Concentrators: A Technology which can Contribute towards Pakistan's Energy Future 99
— *Rizwan Masood*

Naturally Fractured Reservoirs – Yet an Unsolved Mystery 109
— *Muhammad Khurram Zahoor and Muhammad Haris*

Classification of Scientific Publications using Swarm Intelligence 115
— *Tariq Ali , Sohail Asghar, Naseer Ahmed Sajid and Munir Ahmad*

Life Sciences

Evaluation of Relative Efficacy of Hypertonic Saline and Lactated Ringer's Solutions in Experimentally-induced Hypo-volumic Shock in Dogs 127
— *Muhammad A. Zafar, Nemat Ullah, Imran Sarfaraz and Fahad Raza*

Comparative Phycoremediation of Sewage Water by Various Species of Algae 131
— *Farooq Ahmad, Amin U. Khan and Abdullah Yasar*

Physical Sciences

Remarks on (1,2)*-αg-Homeomorphisms 141
— *T. Salai Parkunan, S. Pious Missier, Ravi and A. Pandi*

Seasonal Variability in Virtual Height of Ionospheric F_2 Layer at the Pakistan Atmospheric Region 151
— *Akbar Ali Jilani, M. Ayub Khan Yousuf Zai, Faisal Ahmed Khan Afridi, Khusro Mian and Syed Nazeer Alam*

Extreme Rainfall Incidents over Sindh Province, against Different Return Periods 159
— *Muhammad Ali and M.J. Iqbal*

Properties of Univalent Solution for Complex Fractional Differential Equation 167
— *Rabha W. Ibrahim and Maslina Darus*

Obituary: Prof. Dr. Mustafa Shameel 173

Instructions for Authors 175

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